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Abstract

This paper analyses the consequences of environmental myopia on policy design in a multi-pollutants framework. Focusing on the correlations between aerosols and greenhouse gases, the paper compares abatement and stock targets setting for various cases of environmental myopia. Both cases of lax and stringent regulation, compared to what is socially optimum, may arise. Furthermore, the lax/stringent nature of the policies may evolve over time, so that the time horizon of policy design matters in assessing the impact of environmental myopia.

1 Introduction

It is now accepted that atmospheric pollution issues in general, and climate change in particular, are imputable to a number of interrelated constituents. Consequently, the need for a comprehensive approach to atmospheric pollution is increasingly advocated (Reilly and Richards, 1993; Reilly et al., 1999; Aaheim, 1999; Manne and Richels, 2000; Schmieman, Van Ierland, and Hordijk, 2002; Caplan and Silva, 2005; Yang, 2006; Moslener and Requate, 2007, 2009). Early studies of atmospheric pollution, if they don't deny the existence of multiple, interrelated pollutants, rather assume that they can be treated separately or aggregated to a single measure of pollution. The most recent theoretical analyses of multi-pollutants situations address the current policy approach to climate change management : a fixed set of identified contributors[1] which greenhouse impacts are compared through static, exogenously defined Global Warming Potentials (GWPs) (Moslener and Requate, 2009; De Cara, Debove, and Jayet, 2006). These studies call into question both the choice of listed constituents[2] and the static nature of the "exchange rates", the GWPs (Moslener and Requate, 2007, 2009; De Cara, Debove, and Jayet, 2006). Indeed, pollutants can interact both during the production process, with joint emission from a production activity and interacting abatement technologies (Caplan and Silva, 2005; Moslener and Requate, 2007), and once the pollutants are emitted and accumulate. Indeed, cases of interacting decay processes or joint impact on damage have been documented (Moslener and Requate, 2009; Yang, 2006). Consequently, single-pollutant focused analyses do not fully capture the interactions that develop between pollutants.

Bearing this in mind, the starting point of this paper is the assumption that policy approaches to atmospheric pollution have typically been based on mis-specified environmental models, in the sense that they have not captured all the contributors to the issue, or if they have, that the interactions between the contributors have not been properly described. We refer to these mis-specifications as cases of environmental myopia : the notion of myopia is not linked to the time horizon of policy-making but to the specification of the damage function. The objective of this paper is then to analyse how environmental myopia may lead policy makers to set sub-optimal pollution targets in a multi-pollutants setting.

Different cases of interacting pollutants have been described in the scientific

[1] GreenHouse Gases (GHG) listed under Annex A of the Kyoto Protocol : CO_2, CH_4, N_2O, $PFCs$, HFC and SF_6.

[2] Some "missing" GHGs have been identified, such as tropospheric ozone.

literature (Reilly and Richards, 1993; Reilly et al., 1999) and a few analysed in an economic framework (Schmieman, Van Ierland, and Hordijk, 2002; Caplan and Silva, 2005; Barrett, 2008; Smulders, Daiju, and Cruz, 2009). This paper focuses on the particular example of "*carbon*" and "*sulphur*", as shortcuts for anthropogenic greenhouse gases (GHGs) and aerosols. GHGs are the atmospheric constituents identified as responsible for global warming. They also have detrimental impacts such as oceans' acidification and other ecological disruptions. Aerosols, sulphur in particular, are responsible for a number of atmospheric pollution issues such as acid rains. Their peculiar contribution to the regulation of climate is also increasingly put forward : they have a recognised cooling effect on global temperature. From this property stems the development of climatic geoingeneering or"*the deliberate modification of the climate by means other than by changing the atmospheric concentration of GHGs*"(Barrett, 2008, p.45). A real-life example was provided by the 1991 eruption of Mount Pinatubo, in the Philippines, which induced a decrease of the Earth's surface temperature of about 0.5°C the following year due to the huge amount of sulphur that was injected in the atmosphere during the eruption (Crutzen, 2006). Some recent works (Barrett, 2008; Smulders, Daiju, and Cruz, 2009) provide the economics foundation to include geoengineering into the policy debate. In this paper, we assimilate geoengineering to the reduction of sulphur abatement levels, hence we will not refer to geoengineering *per se* but to sulphur management.

In a first step, we study a basic model where the only type of interaction considered between carbon and sulphur is their combined impact on climate change. We show how not accounting for the cooling impact of sulphur induces a too stringent sulphur management, be it defined in terms of abatement or stock levels. We also discuss the impact on welfare and emphasise the tradeoff between the direct effect of sulphur through acidification and its indirect impact on climate. In a second step, we recognize that, apart from their combined impact on climate regulation, carbon and sulphur are also linked at the production stage. Indeed, it has been documented that sulphur abatement technologies are associated with high levels of GHG emissions (Moslener and Requate, 2007). We show how, in this modified framework, stationary targets are altered, as compared to the previous case; indeed, both cases of lax and stringent carbon regulation may arise. We also put in perspective the importance of the time-horizon of the policy instrument. Indeed, non monotonic accumulation and abatement patterns may arise in such a context, meaning that a lax instrument, in the short run, may become too stringent when the time horizon is extended, or vice-versa.

Section 2 presents the model. Section 3 is dedicated to stationary target setting in the basic case and when a technological interaction is introduced. In Section 4, we illustrate the importance of the policy instrument's time horizon. Section 5 concludes.

2 The Model

Consider a representative polluter whose production activity generates the joint emission of two pollutants : carbon (referred to by subscript 1) and sulphur (subscript 2). Abatement technologies are available for both pollutants and denoted by $a_1(t)$ and $a_2(t)$, so that in the basic case the emission functions for carbon and sulphur are :

$$e_1(t) = \bar{e}_1 - a_1(t), \tag{1}$$

$$e_2(t) = \bar{e}_2 - a_2(t), \tag{2}$$

where $\bar{e}_1$ and $\bar{e}_2$ are positive, unrestricted emission levels. Polluting emissions accumulate in distinct stocks, S_1 and S_2, according to the following dynamics :

$$\dot{S_1}(t) = e_1(t) - \delta_1 S_1(t) \text{ , with : } S_1(0) = S_1^0, \tag{3}$$

$$\dot{S_2}(t) = e_2(t) - \delta_2 S_2(t) \text{ , with : } S_2(0) = S_2^0, \tag{4}$$

where δ_1 and δ_2 are the decay rates of, respectively, carbon and sulphur[3] . The abatement cost and damage functions are quadratic and given by[4] :

$$C(a_1(t), a_2(t)) = \frac{c_1}{2} a_1(t)^2 + \frac{c_2}{2} a_2(t)^2, \tag{5}$$

$$D(S_1(t), S_2(t)) = \frac{d_1}{2} S_1(t)^2 + \frac{d_2}{2} S_2(t)^2 + d_3 T(S_1(t), S_2(t)), \tag{6}$$

where : $T(S_1(t), S_2(t)) = t_1 S_1(t) - t_2 S_2(t)$.

Equation (6) contains three types of damage. The first two terms on its RHS capture the direct damage from the accumulation of carbon and sulphur. The third describes the damage due to climate change, assumed linear (Labriet and Loulou, 2003) in global temperature $T(S_1, S_2)$. The temperature is correlated positively to the stock of carbon and negatively to the stock of sulphur.

In section 3.3, this basic model is extended to account for the fact that the emission and abatement technologies of the different pollutants may not be completely independent (Caplan and Silva, 2005; Moslener and Requate, 2007). Var-

[3] $0 < \delta_1 < 1$ and $0 < \delta_2 < 1$.

[4] Parameters c_1, c_2, d_1, d_2, d_3, t_1 and t_2 are all positive.

ious examples of technological correlations have been documented, among which joint emission (the burning of fossil fuels leads to the emission of CO_2, SO_2, ozone, nitrogen oxide, CO, volatile organic compounds and particulate matters), coarse abatement (absorption technologies simultaneously remove gaseous pollutants and dust from polluting emissions (Caplan and Silva, 2005)) and substitute technologies (sulphur abatement technologies reduce sulphur emissions at the cost of increased carbon emission levels (Moslener and Requate, 2007)). Besides joint emission, already captured in the basic model, we introduce the third property of substitute technologies so that the carbon emission function is modified as follows, with $0 < \alpha < 1$:

$$e_1(t) = \bar{e}_1 - a_1(t) + \alpha a_2(t). \tag{7}$$

Equation (7) illustrates the fact that sulphur abatement increases the level of carbon emissions. Note that this is the only difference between the basic case and the case with technological correlation.

3 Stationary target setting

This section is devoted to the analysis of stationary target setting : we assess how environmental myopia may affect sulphur and carbon management strategies, when they consist in setting a regulatory constraint on either the level of abatement or the level of accumulation that should be respected in the long run (at the steady state).

3.1 Environmental myopia in the carbon/sulphur context

This model allows studying two cases of environmental myopia that may arise in a multi-pollutants setting. The first is the focus on one pollutant. Case D corresponds to a sulphur focused management : climate regulation is not a policy matter in this fictitious case. Case C proposes a carbon focused management : here, the various impacts of sulphur accumulation, in terms of acidification and climate regulation, are overlooked. The second type of environmental myopia we analyse in this paper arises when all the pollutants are under management, but their interactions are not properly described. Here, the cooling impact of sulphur is not recognised by the regulator (case B).

Table 1 illustrates the programs of the regulators in each of the 4 cases studied in the paper. Case A refers to the socially optimal program that would be followed by a benevolent and fully informed regulator.

Case	Control(s)	Objective function to be min.	State equation(s)
A	$a_1(t)$, $a_2(t)$	$C(a_1(t), a_2(t))+$ $D(S_1(t), S_2(t), T(S_1(t), S_2(t)))$	$\dot{S}_1(t) = e_1(t) - \delta_1 S_1(t)$ $\dot{S}_2(t) = e_2(t) - \delta_2 S_2(t)$
B	$a_1(t)$, $a_2(t)$	$C(a_1(t), a_2(t))+$ $D(S_1(t), S_2(t), T(S_1(t)))$	$\dot{S}_1(t) = e_1(t) - \delta_1 S_1(t)$ $\dot{S}_2(t) = e_2(t) - \delta_2 S_2(t)$
C	$a_1(t)$	$C(a_1(t), 0) + D(S_1(t), T(S_1(t)))$	$\dot{S}_1(t) = e_1(t) - \delta_1 S_1(t)$
D	$a_2(t)$	$C(0, a_2(t)) + D(S_2(t))$	$\dot{S}_2(t) = e_2(t) - \delta_2 S_2(t)$

Table 1: Cases analysed : social optimum and various environmental myopia.

For each case, we assume that the policy-maker, at the beginning of the time horizon, decides on an abatement (resp. stock) target that would have to be respected in the long-run. This target is set equal to the steady state abatement (resp. stock) level obtained from solving his minimization program. The comparison of the various myopic strategies with the socially optimal solution leads to the formulation of Propositions 1 and 3.

After studying the basic case where the only correlation captured is the joint impact on climate regulation (Section 3.2), we introduce the technological correlation (Section 3.3) : this significantly alters the results.

3.2 The basic case : a simple model of carbon/sulphur interaction

Proposition 1. *With damage, cost and accumulation functions as described in Equations* (1)-(6)*, environmental myopia don't affect carbon management but lead to too stringent sulphur management compared to what is socially optimal.*

Proof. The analytical expressions of abatement and stock levels are provided in Appendix 2, and the general derivation method is presented in Appendix 1. The targets compare as follows :

$$S_{1\infty}^A = S_{1\infty}^B = S_{1\infty}^C \text{ and } a_{1\infty}^A = a_{1\infty}^B = a_{1\infty}^C, \tag{8}$$

$$S_{2\infty}^A > S_{2\infty}^B = S_{2\infty}^D \text{ and } a_{2\infty}^A < a_{2\infty}^B = a_{2\infty}^D. \tag{9}$$

Equations (8) and (9) directly lead to the formulation of Proposition 1. $\square$

What Proposition 1 states is that under the assumptions of the basic model, myopic management can differ from socially optimal decisions only regarding sulphur management, which turns out to be too stringent. Indeed, when carbon altogether, or the cooling impact of sulphur only, are not accounted for, the beneficial impact that sulphur may have in terms of climate mitigation is not perceived. Consequently the incentive to abate sulphur is greater than under the socially optimal management. The consideration of the technological correlation will alter this result (Section 3.3). Carbon management, on the other hand, is not affected by the types of environmental myopia addressed in the paper.

Next we compare how myopic and socially optimal strategies differ in terms of actual welfare, by comparing the actual environmental and abatement costs, rather than those perceived by the myopic regulator. To do so, we compute the levels of abatement and stock of the pollutant not under direct management, sulphur in case C and carbon in case D. No damage being associated to the pollutant's emission or accumulation by the policy maker, the resulting level of abatement is nul, and the steady state stock level easily derived as emissions are constant and equal to their unrestricted levels. For illustrative purposes, they compare to the socially optimal levels in the following manner :

$$S_{1\infty}^A < S_{1\infty}^D \ , \ a_{1\infty}^A > a_{1\infty}^D = 0 \ , \ a_{2\infty}^A > a_{2\infty}^C = 0, \tag{10}$$

$$S_{2\infty}^A - S_{2\infty}^C < 0 \Leftrightarrow \frac{d_3 t_2}{d_2} < \frac{\bar{e_2}}{\delta_2}. \tag{11}$$

Unsurprisingly, a regulator focusing on sulphur induces too much carbon accumulation (Equation (10)). Also, a carbon-focused regulator doesn't abate for sulphur. The resulting impact on the stock of sulphur, as compared to the social optimum, depends on the relative "weight" of the direct and indirect damage due to sulphur, hence how these effects are accounted for in the social optimum (Equation (11)). The higher the damage associated to the stock of carbon in itself, the higher the chance that the myopic stock of sulphur is higher than the socially optimal one.

Proposition 2. *With damage, cost and accumulation functions as described in Equations* (1)-(6)*, actual welfare levels compare differently depending on the value of the ratio* $t_2 d_3/d_2$ *:*

$$W_D < W_B < W_C < W_A \textit{ for } m_1 < t_2 d_3/d_2, \tag{12}$$

$$W_D < W_B < W_A < W_C \text{ for } m_2 < t_2 d_3/d_2 < m_1, \tag{13}$$

$$(W_D, W_A) < W_B < W_C \text{ for } m_3 < t_2 d_3/d_2 < m_2, \tag{14}$$

$$(W_D, W_A) < W_C < W_B \text{ for } t_2 d_3/d_2 > m_3, \tag{15}$$

where m_1, m_2 and m_3 are given in the proof.

Proof. Noting $W_j = -\left[C(a_{1j}^{\infty}, a_{2j}^{\infty}) + D(S_{1j}^{\infty}, S_{2j}^{\infty})\right]$ with $j \in \{A, B, C, D\}$, we obtain the following relations :

$$W_B - W_D > 0, \tag{16}$$

$$W_A - W_C > 0 \Leftrightarrow \frac{d_3 t_2}{d_2} > m_1 = \frac{\bar{e_2}}{\delta_2}, \tag{17}$$

$$W_A - W_B > 0 \Leftrightarrow \frac{d_3 t_2}{d_2} > m_2 = \frac{\bar{e_2}}{\delta_2 + \frac{\delta_2^2}{2r} + \frac{d_2}{2c_2 r}}, \tag{18}$$

$$W_B - W_C > 0 \Leftrightarrow \frac{d_3 t_2}{d_2} < m_3 = \frac{\bar{e_2}}{\delta_2}\frac{1}{2}\left(1 + \frac{1}{1 + \frac{\delta_2}{r} + \frac{d_2}{c_2 r \delta_2}}\right), \tag{19}$$

Note that the sign of $W_A - W_D$ is undetermined[5]. Also, it can be easily verified that $m_1 > m_2 > m_3$. □

Proposition 2 highlights how the relative values of direct damage and indirect avoided climate-related damage from sulphur accumulation can induce very different situations in terms of actual welfare. It appears that when the cooling effect is strong (Equation 12), case A generates the highest actual welfare level. However, a decrease of the cooling effect compared to the direct sulphur effect may induce myopic management (cases B and C) to produce higher welfare levels than the socially optimal management. Indeed, both low abatement levels and reduced stocks increase welfare. For instance, in case C, the level of sulphur abatement is nul, reducing total costs.

Considering only post-emission interactions, it appears that sulphur management strategies are unambiguously too stringent compared to what would be optimal. Such a statement is in line with studies that point out that past aerosols emissions have constituted an unrecognised "cooling device" so that current strategies to mitigate their emissions may lead to an unexpected temperature rise (Andreae, Jobes, and Cox, 2005)[6].

[5]The analytical expression is tedious, hence not presented in the paper for the sake of clarity.

[6]An even stronger statement is that "*Incomplete consideration of aerosols in current climatic models have led to underestimation of the true climate sensitivity*" (Andreae, Jobes, and Cox, 2005, p. 1190).

3.3 A model with two types of correlations

This section extends the previous analysis to the case where carbon and sulphur are linked not only after they have been emitted, but also during the production stage.

Proposition 3. *When the technological correlation is introduced as in Equation* (7)*, myopic strategies always induce too stringent sulphur management. However, the nature of the myopia is important in assessing the impact on carbon management. While in case B, a target setting strategy is always either too stringent (for abatement) or too lax (for stock), in cases C and D target setting may be either too lax or too stringent (for both stock and abatement).*

Proof. Appendix 3 contains the analytical expressions of steady state abatement and stock levels, which compare as follows :

$$S_{1\infty}^{B} > S_{1\infty}^{A} \ , \ a_{1\infty}^{B} > a_{1\infty}^{A}, \tag{20}$$

$$S_{2\infty}^{A} > S_{2\infty}^{B} > S_{2\infty}^{D} \ , \ a_{2\infty}^{D} > a_{2\infty}^{B} > a_{2\infty}^{A}. \tag{21}$$

Furthermore, the sign of $(S_{1\infty}^{C} - S_{1\infty}^{A})$, which is the same as the sign of $(a_{1\infty}^{C} - a_{1\infty}^{A})$, depends on the value of the ratio $t_2 d_3 / d_2$: it is positive for low values of the ratio and negative for high values (see Appendix 3). □

Impact of climate-related damage misspecification : cases A-B. When the cooling effect of SO_2 is not accounted for, then, compared to the social optimum, too much carbon and too few sulphur are accumulated and both carbon and sulphur abatement levels are too high. The fact that, at the same time, carbon abatement and stock levels are higher than in the socially optimal case can be explained by the role played by the technological correlation : a higher sulphur abatement level is associated to higher carbon emissions, hence carbon stock. Indeed, the stronger the correlation, the higher the difference between steady state carbon stocks. The fact that more carbon is accumulated when the cooling impact is not accounted for might seem surprising. Indeed one would expect the sulphur effect to constitute a possibility to accumulate more carbon because part of its damage is offset by the accumulation of sulphur. However, at the steady state, the direct effect from the technological correlation dominates the indirect effect. Note that, in the absence of technological correlation, steady state carbon stocks are equal (section 3.2). Consequently, a two-stock myopic policy based on stock targets would be at the same time too lax for carbon and too stringent for sulphur. Conversely, a myopic policy based on abatement targets would be too stringent for both pollutants, due to the technological correlation.

Impact of focusing on one stock : cases A-C or D. The impact of having the decision maker focusing on sulphur management only is non ambiguous : the abatement level is higher, and the resulting stock lower, than what is socially optimal. Not accounting for carbon has a double impact on sulphur management : first, a greater incentive to abate, because the accumulation of sulphur is not perceived as having a cooling impact on climate, and, second, a lower incentive to avoid abating because abatement has no perceived impact on carbon emission. Consequently, sulphur is more abated when focus is on this pollutant only, assuming, of course, that damage are evaluated at the same level in all cases. When a decision maker focuses on carbon management only, the resulting impact on target setting depends on the value of the ratio $t_2 d_3/d_2$. For relatively low values of the ratio, which indicate that the direct effect of sulphur via acidification dominates the indirect climate effect, carbon management is too lax (regarding stock) and too stringent (regarding abatement). Conversely, when the indirect effect dominates the direct impact, stock targets are too high, and abatement targets too low, compared to the socially optimal solution.

To sum up, while both types of environmental myopia induce too stringent sulphur management, their impact on carbon management depends on the type of myopia and on the relative importance of the direct and indirect environmental impacts of sulphur. Furthermore, both types of myopia affect carbon abatement and stock accumulation in opposite ways, so that in the same situation a myopic regulator would be labelled as too stringent regarding stock target setting and too lax for abatement targets.

When the high energy consuming characteristic of sulphur is accounted for, and expressed as an indirect impact of sulphur abatement on carbon emission levels, myopic sulphur management remains unambiguously too stringent compared to what is socially optimal. Then, if we accept that the actual current situation is that of the climate-related impact of sulphur not being recognised in climate mitigation strategies, then it appears that long term abatement targets of both carbon and sulphur are too stringent.

4 Short term targets : impact of the time horizon of the policy

The analyses conducted in Section 3 concerned the setting of targets in the long run. However, atmospheric pollution management strategies are designed within the political time framework, on a shorter term, so that the assumption of stationary targets may appear too strong. Consequently, in this section, we address the setting of abatement and pollution stock targets along the whole time horizon, and in particular on the short term. Our analysis shows that there are various sets of conditions under which the relative positions of the myopic and optimal accumulation paths evolve over time. In other words, a myopic management may be too lax (stringent) on the short term, and become too stringent (lax) on the longer term. For illustrative purposes, the remainder of the paper is devoted to carbon stock targets.

4.1 Non standard carbon accumulation paths

Previous studies have shown that in a setting with multiple pollutants, non-standard accumulation patterns could arise (Moslener and Requate, 2007, 2009; Baumgärtner, Jöst, and Winkler, 2009) in contrast with the single pollutant setting, where accumulation paths are typically monotonous. This difference in accumulation patterns proves to be conducive, under certain conditions, to an evolution of the relative position of the stock targets, under myopic and socially optimal management, over time. For the sake of illustration, we focus on carbon, by comparing the polar cases of optimal (case A) and "*totally*" myopic (case C) management[7].

Proposition 4. *The carbon accumulation path may exhibit the following properties in case A (K, ρ_1 and ρ_2 are defined in the proof) :*

A if $K < \frac{\rho_1}{\rho_2}$, then $S_1(t)$ is monotonous and exhibits a unique curvature,

B if $\frac{\rho_1}{\rho_2} < K < 1$, then $S_1(t)$ is monotonous and exhibits an inflection point,

C if $K > 1$, then $S_1(t)$ is non monotonous.

Proof. Refer to Appendix 4 for the demonstration. K is defined as follows :

$$K = \frac{\rho_2}{\rho_1} \frac{\Delta S_2^A / \Delta S_1^A - \rho_{1S_2}/\rho_{1S_1}}{\Delta S_2^A / \Delta S_1^A - \rho_{2S_2}/\rho_{2S_1}} > 1, \tag{22}$$

[7]Note that the application to sulphur stock targets is straightforward. Concerning the setting of emission, or abatement, targets, the methodology is the same, but one has to account for the fact that the initial levels are not given, hence not necessarily equal for cases A and C, even if the initial stock levels are equal.

with $\Delta S_i = S_{i\infty} - S_i(0)$, ρ_1 and ρ_2 the negative eigenvalues that guide the accumulation paths (with $|\rho_2| > |\rho_1|$ hence ρ_2 is labelled as the fast eigenvalue and ρ_1 the slow one) and ρ_{iS_j} are the components of the associated eigenvectors (see Appendix 4). $\square$

Equation (22) shows that the value of K depends on 3 ratios :

- ρ_2/ρ_1 : this ratio is necessarily positive, as the chosen eigenvalues are negative to comply with the transversality condition. A high ratio may result from either a high $|\rho_2|$ or a low $|\rho_1|$, indicating that the slope of the curve is steeper in the short run (where the fast eigenvalue plays a greater role) than in the long run (where the slow eigenvalue matters more);

- ρ_{iS_2}/ρ_{iS_1}, $i = \{1, 2\}$: these ratios compare how each eigenvalue affects the accumulation paths of sulphur and carbon in the short run ($i = 2$) and in the long run ($i = 1$). In particular, for $i = 1$ the sign of this ratio informs of whether the accumulation paths approach their respective steady states both from below or above ($\rho_{iS_2}/\rho_{iS_1} > 0$) or one from above and one from below ($\rho_{iS_2}/\rho_{iS_1} < 0$);

- $\Delta S_2^A/\Delta S_1^A$: the sign of this ratio depends on the relative values of the steady state carbon and sulphur stocks compared to their respective initial levels. In particular, when carbon and sulphur are symmetric with respect to the location of the steady state compared to the initial level, then the ratio is positive. Furthermore, this ratio increases, in absolute value, with a high $|\Delta S_2|$ and a low $|\Delta S_1|$. Consequently, the behavior of carbon accumulation depends explicitely on the stock of sulphur.

The first two ratios are independant of the initial conditions. Consequently, with the same set of parameters, the relative position of the initial carbon and sulphur stocks may induce the carbon stock path to be of very different nature.

4.2 Illustrative examples of a change of the relative position of the myopic and optimal carbon paths

In this section, we analyse two cases where myopic target strategies change of nature (too lax/stringent compared to the social optimum) over time. Results 1 and 2 sum up theses cases. Result 3 illustrates a situation where, if the nature of the policy doesn't change over time, the discrepancy between targets evolves greatly over time[8].

[8]Appendix 5 contains the demonstrations of Results 1 -3.

This section isn't aimed at being exhaustive. Rather, we point out some situations of interest that may lead to very different conclusions regarding the impact of environmental myopia, depending on the time scale in use.

The myopic and socially optimal carbon stock paths can be expressed as follows :

$$S_1(t)^C = C_1^C r_{1S_1} e^{r_1 t} \tag{23}$$

$$S_1(t)^A = \sum_{i=1}^{2} C_i \rho_{iS_1} e^{\rho_i t} \tag{24}$$

Result 1. *If the initial stock of carbon is lower than both myopic and optimal stationary levels, the following conditions are sufficient to induce too stringent short term targets and too lax stationary targets :*

$$\frac{\rho_1}{\rho_2} < K < 1 \ , \ C_1 \rho_1 \rho_{1S_1} > 0 \ \textit{and} \ C_2 \rho_2 \rho_{2S_1} < 0,$$

under the assumption that $|\rho_2| > |\rho_1|$.

[Figure 1 about here]

The context illustrated in Result 1 is that of initial carbon and sulphur stocks below both optimal and myopic steady state levels. Under the conditions stated in Result 1, both carbon paths are increasing over time, though the initially low level of sulphur emissions induces a lower initial speed of accumulation in the optimal case. Hence, on the short run, a myopic regulator may be assessed as too stringent compared to a benevolent regulator. However, as the stock of sulphur increases, the socially optimal level of carbon emissions tends to increase. Consequently, on the long run, myopic carbon stock targets become too stringent. This is illustrated in Figure 1, which was obtained for parameters' values described in Appendix 6, in particular for an initial stock of sulphur below its steady state value.

Result 2. *If the initial stock of carbon is at an intermediary level between the myopic and optimal stationary levels, the following conditions are sufficient to induce too lax short term targets and too stringent stationary targets :*

$$K > 1 \ \textit{and} \ C_1 \rho_1 \rho_{1S_1} + C_2 \rho_2 \rho_{2S_1} < C_1^C r_1 r_{1S_1} < 0,$$

under the assumption that $|\rho_2| > |\rho_1|$.

[Figure 2 about here]

Result 2 describes a situation where the initial stock of carbon lies between the optimal and myopic steady state levels. Consequently, the myopic carbon stock decreases over time and, under some conditions on the initial stocks, the socially optimal carbon stock may exhibit a non monotonic accumulation pattern. Figure 2 illustrates a case where the initial sulphur stock is also intermediary (see Appendix 6). We observe that during a first phase, the socially optimal stock path decreases, due to a strong increase of the level of carbon abatement, followed by a strong increase of energy-consuming sulphur abatement. Myopic stock targets are too lax. The effect is then reversed and the myopic target become too stringent as the optimal carbon stock reaches a strongly increasing phase while the myopic path continues decreasing. Hence, the myopic strategy will be assessed as too stringent on the short run and too lax on the long run.

Result 3. *If the initial stock of carbon is above both myopic and optimal stationary levels, the following conditions induce the type of pattern shown in Figure 3, namely targets increasingly stringent over time after a short period where myopic and optimal paths are quite close :*

$$K > 1 \ , \ C_1\rho_1\rho_{1S_1} + C_2\rho_2\rho_{2S_1} < 0 \ \textit{and} \ C_1^C r_1 r_{1S_1} < 0,$$

under the assumption that $|\rho_2| > |\rho_1|$.

[Figure 3 about here]

To complete the analysis, Result 3 addresses the case of an initial stock of carbon located above both stationary levels. Figure 3 provides an illustration where the initial stock of sulphur is below the steady states levels. Once again, the myopic carbon stock is decreasing; while the socially optimal carbon stock increases after a short decreasing period. This is due to the fact that sulphur abatement levels increase at the expense of higher carbon emissions. Consequently, if the myopic strategy is always too lax, the extent to which it exceeds the socially optimal strategy varies over time.

5 Concluding remarks

In this paper we propose a framework to analyse carbon and sulphur management in a multi-pollutants setting. We allow for the pollutants to interact both during the production process, to illustrate the links between sulphur abatement technologies and carbon emissions, and after the production process, to capture the joint impact that carbon and sulphur have on global temperature, hence climate change.

Our analysis of the basic model, where the combined impact on climate change is the only type of interaction considered between carbon and sulphur, shows how not accounting for the cooling impact of sulphur induces a too stringent sulphur management, be it defined in terms of abatement or stock levels. We also discuss the impact on welfare and emphasise the tradeoff between the direct effect of sulphur through acidification and its indirect impact on climate. When a more realistic model is used, that recognizes that apart from their combined impact on climate regulation carbon and sulphur are also linked during the production process, stationary targets are altered, as compared to the previous case. Indeed, both cases of lax and stringent carbon regulation may arise. We also put in perspective the importance of the time-horizon of the policy instrument. Non monotonic accumulation and abatement patterns may arise in a multi-pollutants context, meaning that a lax instrument, in the short run, may become too stringent when the time horizon is extended (or vice-versa).

From our analysis, it appears that whatever the type of environmental myopia defined in this paper, sulphur management is too stringent because its cooling impact on climate is not recognised. The importance of aerosols in the mitigation of climate change is now recognised, to such an extent that some authors expect that the increasingly stringent aerosols mitigation strategies currently implemented may have the "unexpected" impact of contributing to global warming by decreasing the cooling impact of sulphur (Andreae, Jobes, and Cox, 2005). We also put in perspective that two parameters are crucial in the assessment of how myopic management compares to socially optimal decisions : the initial stock levels of both carbon and sulphur, in particular their positions compared to the stationary levels, and the time frame considered. Indeed, we illustrate that due to the non standard accumulation patterns that may arise in a multi-stocks setting, the nature of myopic targets (lax/stringent) may evolve over time. The questions of where we currently stand in terms of aerosols and GHG accumulation and the definition of "optimal" levels, are still debated.

We chose not to discuss the origin of the environmental myopia analysed in the paper, and rather take them as given in order to assess how they could affect a certain type of policy making. The particular type of policy we focus on is the setting of regulatory constraints, expressed in terms of abatement or stock levels, no to be exceeded at various time scales (steady state and shorter term). This one-time policy strategy allows us to disregard issues relating to the time-consistency of the instrument. At least two explanations to the recourse to environmental myopia by policy-makers could be provided in this context.

First, the lack of scientific knowledge that leads the policy-maker to make use of restrictive models of the environment. In this case, the model could be extended to allow for more refined models of scientific uncertainty, for instance encompassing the notion of ambiguity. Second, the recourse to restricted environmental models could result from a strategic decision-making process. Capturing this notion would necessitate an extension of the model to at least two players and maybe the introduction of a regional aspect to the depiction of the environmental issues, aspect which is absent from this present model. Despite its necessarily simplifying assumptions, we believe that this model puts in perspective important consequences that past atmospheric pollution management may have had, and perhaps more importantly those that future strategies may have if the interactions between atmospheric pollutants are not accounted for in a more comprehensive manner.

Appendix 1 : Saddle point property of the steady state in a two stocks model

The saddle point property of the steady state is shown in the social optimum case and applies to the more restrictive case B. A benevolent regulator has the following program :

$$\min_{a_1(t),a_2(t)} \int_{t=0}^{+\infty} e^{-rt}\left[C(a_1(t),a_2(t)) + D(S_1(t),S_2(t),T(S_1(t),S_2(t))\right] dt, \quad \text{(P.A.)}$$

$$\text{s.t. } \dot{S}_1(t) = \bar{e}_1 - a_1(t) + \alpha a_2(t) - \delta_1 S_1(t),$$

$$\text{and } \dot{S}_2(t) = \bar{e}_2 - a_2(t) - \delta_2 S_2(t).$$

The time subscript is dropped in the remainder of the paper. Note λ and μ the co-state variables attached to carbon and sulphur stocks. The Hamiltonian is then :

$$H^A = C(a_1,a_2)+D(S_1,S_2,T(S_1,S_2))+\lambda[\bar{e}_1-a_1+\alpha a_2-\delta_1 S_1]+\mu[\bar{e}_2-a_2-\delta_2 S_2]. \quad (25)$$

The first order conditions, derived from the Hamiltonian, are given by :

$$\frac{\partial C}{\partial a_1} - \lambda = 0 \quad (26)$$

$$\frac{\partial C}{\partial a_2} + \alpha\lambda - \mu = 0 \quad (27)$$

$$\dot{\lambda} - r\lambda = -[\frac{\partial D}{\partial S_1} + \frac{\partial D}{\partial T}\frac{\partial T}{\partial S_1} - \delta_1\lambda] \tag{28}$$

$$\dot{\mu} - r\mu = -[\frac{\partial D}{\partial S_2} + \frac{\partial D}{\partial T}\frac{\partial T}{\partial S_2} - \delta_2\mu] \tag{29}$$

together with transversality conditions and positivity constraints on a_1 and a_2. These first-order conditions can be rearranged to form the Modified Hamiltonian Dynamic System, expressed under matrix form as follows :

$$\begin{pmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{\lambda} \\ \dot{\mu} \end{pmatrix} = J. \begin{pmatrix} S_1 \\ S_2 \\ \lambda \\ \mu \end{pmatrix} + C,\ J = \begin{pmatrix} -\delta_1 & 0 & k_1 & k_2 \\ 0 & -\delta_2 & k_2 & k_3 \\ -d_1 & 0 & r+\delta_1 & 0 \\ 0 & -d_2 & 0 & r+\delta_2 \end{pmatrix},\ C = \begin{pmatrix} \bar{e_1} \\ \bar{e_2} \\ -t_1 d_3 \\ t_2 d_3 \end{pmatrix}$$

$$\text{with } k_1 = -\frac{1}{c_1} - \frac{\alpha^2}{c_2} < 0\ ,\ k_2 = \frac{\alpha}{c_2} > 0 \text{ and } k_3 = -\frac{1}{c_2} < 0.$$

Two conditions are necessary and sufficient to ensure the saddlepoint property (Dockner, 1985) :

$$\textbf{(C1)} : M < 0 \text{ and } \textbf{(C2)} : 0 < \det J \leq (\frac{M}{2})^2,$$

$$\text{where } M = \begin{vmatrix} \frac{\partial \dot{S}_1}{\partial S_1} & \frac{\partial \dot{S}_1}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial S_1} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{vmatrix} + \begin{vmatrix} \frac{\partial \dot{S}_2}{\partial S_2} & \frac{\partial \dot{S}_2}{\partial \mu} \\ \frac{\partial \dot{\mu}}{\partial S_2} & \frac{\partial \dot{\mu}}{\partial \mu} \end{vmatrix} + 2\begin{vmatrix} \frac{\partial \dot{S}_1}{\partial S_2} & \frac{\partial \dot{S}_1}{\partial \mu} \\ \frac{\partial \dot{\lambda}}{\partial S_2} & \frac{\partial \dot{\lambda}}{\partial \mu} \end{vmatrix}.$$

Demonstration of (C1). Due to the negativity of k_1 and k_3, it can be easily verified that the following expression is negative :

$$M = -\delta_1(r+\delta_1) + d_1 k_1 - \delta_2(r+\delta_2) + d_2 k_3 < 0.$$

Demonstration of (C2). The positivity of the determinant of the Jacobian is straightforward :

$$\det J = \delta_2\delta_1 r(r+\delta_2+\delta_1) + d_1 d_2(k_1 k_3 - k_2^2) - d_1 k_1(\delta_2+r)\delta_2 - d_2 k_3(r+\delta_1)\delta_1 > 0.$$

Then, after some rearrangements, we obtain the following, which ensures that the second part of **(C2)** holds :

$$(\frac{M}{2})^2 - \det J = \frac{1}{4}(-\delta_1(r+\delta_1) + d_1 k1 + \delta_2(r+\delta_2) - d_2 k_3)^2 + d_1 d_2 k_2^2 > 0.$$

Consequently, the steady state has the saddle point property, whether the technological correlation applies or not.

Appendix 2 : Steady state abatement and stock levels - basic model

The steady state stock and abatement levels are obtained after recognizing that at the steady state, the following relation holds : $Z^{\infty} = -J^{-1}.C$ where Z^{∞} stands for the matrix with typical elements v^{∞}, $v \in \{S_1, S_2, \lambda, \mu\}$. Also, $a_1 = \frac{\lambda}{c_1}$ and $a_2 = \frac{\mu}{c_2}$, $k_1 = -\frac{1}{c_1}$ and $k_2 = -\frac{1}{c_2}$:

$$S_{1\infty}^{A} = \frac{\bar{e_1}(r+\delta_1) + k_1 d_3 t_1}{\delta_1(r+\delta_1) - d_1 k_1} \ , \ a_{1\infty}^{A} = \frac{1}{c_1}\frac{d_1\bar{e_1} + \delta_1 d_3 t_1}{\delta_1(r+\delta_1) - d_1 k_1},$$

$$S_{2\infty}^{A} = \frac{\bar{e_2}(r+\delta_2) + k_3 d_3 t_2}{\delta_2(r+\delta_2) - d_2 k_3} \ , \ a_{2\infty}^{A} = \frac{1}{c_2}\frac{d_2\bar{e_2} - \delta_2 d_3 t_2}{\delta_2(r+\delta_2) - d_2 k_3},$$

$$S_{1\infty}^{C} = \frac{c_1\bar{\mathbf{e_1}}(r+\delta_1) - \mathbf{t_1} d_3}{c_1\delta_1(r+\delta_1) + d_1} \ , \ a_{1\infty}^{C} = \frac{\bar{\mathbf{e_1}} d_1 + \delta_1 \mathbf{t_1} d_3}{c_1\delta_1(r+\delta_1) + d_1},$$

$$S_{2\infty}^{D} = \frac{c_2\bar{\mathbf{e_2}}(r+\delta_1)}{c_2\delta_2(r+\delta_2) + d_2} \ , \ a_{2\infty}^{D} = \frac{\bar{\mathbf{e_2}} d_2}{c_2\delta_2(r+\delta_2) + d_2}.$$

To get the values in case B, just set $t_2 = 0$ in the expressions obtained for case A.

Appendix 3 : Steady state stock and abatement levels - model with technological interaction

Following the same methodology as in Appendix 2, the steady state abatement and stock levels in cases A and B are as follows, considering $a_1 = \frac{\lambda}{c_1}$, $a_2 = \frac{\mu-\alpha\lambda}{c_2}$; with $k_1 = -\frac{1}{c_1} - \frac{\alpha^2}{c_2}$, $k_2 = \frac{\alpha}{c_2}$ and $k_3 = -\frac{1}{c_2}$, and $DEN = \delta_1\delta_2(r+\delta_1)(r+\delta_2) - k_1 d_1 \delta_2(r+\delta_2) - k_3 d_2 \delta_1(r+\delta_1) - d_1 d_2(k_2^2 - k_1 k_3)$:

$$\begin{aligned} S_{1\infty}^{A}.DEN = {} & \bar{\mathbf{e_1}}[\delta_2(r+\delta_2)(r+\delta_1) - d_2 k_3 (r+\delta_1)] + \mathbf{t_1} d_3[k_1\delta_2(r+\delta_2) \\ & + d_2(k_2^2 - k_1 k_3)] + [\bar{\mathbf{e_2}} d_2 - \mathbf{t_2} d_3 \delta_2] k_2 (r+\delta_1), \end{aligned}$$

$$\begin{aligned} S_{2\infty}^{A}.DEN = {} & \bar{\mathbf{e_2}}[\delta_1(r+\delta_1)(r+\delta_2) - d_1 k_1 (r+\delta_2)] + \mathbf{t_2} d_3[-k_3\delta_1(\delta_1+r) \\ & + d_1(k_1 k_3 - k_2^2] + [\bar{\mathbf{e_1}} d_1 - \mathbf{t_1} d_3 \delta_1] k_2 (r+\delta_2), \end{aligned}$$

$$a_{1\infty}^{A}.c_1.DEN = [\bar{\mathbf{e_1}} d_1 + \mathbf{t_1} d_3 \delta_1][\delta_2(r+\delta_2) - d_2 k_3] + d_1 k_2[\bar{\mathbf{e_2}} d_2 - \mathbf{t_2}\delta_2 d_3],$$

$$a_{2\infty}^A.c_2.DEN = [\mathbf{\bar{e_1}}d_1 + \mathbf{t_1}d_3\delta_1][k_2d_2 - \alpha(\delta_2(r+\delta_2) - d_2k_3)] +$$
$$[\mathbf{\bar{e_2}}d_2 - \mathbf{\bar{t_2}}\delta_2 d_3][\delta_1(r+\delta_1) - d_1(k_1+\alpha k_2)],$$

To compare target setting between cases A and B, it suffices to study the sign of the coefficients attached to t_2, as not accounting for the impact of sulphur on $T(S_1, S_2)$ is what characterises the difference between the two programs. Consequently, the impact of an environmental myopia of type B is given by :

$$S_{1\infty}^A - S_{1\infty}^B = -t_2 d_3 \frac{k_2\delta_2(r+\delta_1)}{DEN} < 0,$$

$$S_{2\infty}^A - S_{2\infty}^B = t_2 d_3 \frac{d_1(k_1k_3 - k_2^2) - k_3\delta_1(\delta_1 + r)}{DEN} > 0, \text{ since } k_1k_3 - k_2^2 > 0,$$

$$a_{1\infty}^A - a_{1\infty}^B = -t_2 d_3 \frac{d_1 k_2 \delta_2}{c_1 DEN} < 0,$$

$$a_{2\infty}^A - a_{2\infty}^B = t_2 d_3 \frac{\delta_2[d_1(\alpha k_2 + k_1) - \delta_1(r+\delta_1)]}{c_2 DEN} < 0 \text{ , as } \alpha k_2 + k_1 = -\frac{1}{c_1} < 0.$$

Comparing cases A-D, **sulphur targets** differ between socially optimal and myopic management in the following manner:

$$S_{2\infty}^A - S_{2\infty}^D = \frac{\alpha(r+\delta_1)c_1[\bar{e_1}d_1 + t_1d_3] + t_2d_3[\delta_1(r+\delta_1)c_1 + d_1]}{D2} + \frac{\bar{e_2}c_1d_1d_2\alpha^2(r+\delta_2)}{D3} > 0$$

$$a_{2\infty}^A - a_{2\infty}^D = -\delta_2[S_{2\infty}^A - S_{2\infty}^D] < 0$$

as : $D1 = D2[\delta_1(r+\delta_1)c_1c_2 + d_1(c_2 + \alpha^2 c_1)]$, $D3 = D2c_1[\delta_2(r+\delta_2)c_2 + d_2]$ and $D2 = \frac{DEN}{c_1c_2}$.

Results are less clear-cut for **carbon targets**. Indeed, we obtain :

$$S_{1\infty}^A - S_{1\infty}^C = \frac{(r+\delta_1)[d_2k_2(\bar{e_1}d_1 + \delta_1t_1d_3) + (\bar{e_2}d_2 - t_2d_3\delta_2)(\delta_1(r+\delta_1) - d_1k_1)]}{DEN\delta_1[(r+\delta_1) - d_1k_1]}$$

$$S_{1\infty}^A - S_{1\infty}^C > 0 \Leftrightarrow \frac{t_2d_3}{d_2} < \frac{\bar{e_2}}{\delta_2} + \frac{k_2}{\delta_2}\frac{\bar{e_1}d_1 + \delta_1t_1d_3}{\delta_1(r+\delta_1) - d_1k_1}$$

$$a_{1\infty}^A - a_{1\infty}^C = \frac{d_1}{c_1}\frac{S_{1\infty}^A - S_{1\infty}^C}{r+\delta_1}$$

Setting $t_2 = 0$ in the above expression easily provides the ranking for case B : $S_{1\infty}^B > S_{1\infty}^C$ and $a_{1\infty}^B > a_{1\infty}^C$. Accounting for the cooling effect of sulphur may reverse these signs, under a condition on t_2.

Appendix 4 : non standard carbon stock paths

Denote $\rho_{i\phi}$ the components of the eigenvectors associated to the negative eigenvalues ρ_i, $i \in \{1,2\}$, of the Jacobian matrix and C_i constants determined by the initial conditions. The following applies (see, for instance, Moslener and Requate (2007) :

$$S_1^A(t) = S_{1\infty}^A + \sum_{i=1}^{2} C_i^A \rho_{iS_1} e^{\rho_i t}. \tag{30}$$

C Conditions for non monotonic paths. The monotony properties are derived from the study of dS_1/dt :

$$\frac{dS_1^A(t)}{dt} = \sum_{i=1}^{2} \rho_i C_i^A \rho_{iS_1} e^{\rho_i t}. \tag{31}$$

Equation (31) shows that the time-derivative is composed of two terms, so that the monotony of the accumulation path may change over time. Non monotonic patterns can be ruled out if $\exists i$ such that $\rho_{iS_1} = 0$; indeed if there is only one eigenvector with a component in the S_1 direction, then $S_1(t)$ can only be monotonic[9]. When the two eigenvectors have components in the S_1 direction, then a non-monotonic $S_1(t)$ path exists if there is a strictly[10] positive t^{S_1} such that $\dot{S_1}(t) = 0$. After some rearrangement, we obtain :

$$t^{S_1} = \frac{1}{\rho_1 - \rho_2} \ln \left[-\frac{C_2 \rho_2 \rho_{2S_1}}{C_1 \rho_1 \rho_{1S_1}} \right]. \tag{32}$$

Without loss of generality, assume $|\rho_2| > |\rho_1|$, so that $\rho_1 - \rho_2 > 0$. Then the property of non-monotony depends on the positivity of the ln term, which is equivalent to the following condition, where $\Delta S_i = S_{i\infty} - S_{i0}$:

$$K = -\frac{C_2 \rho_2 \rho_{2S_1}}{C_1 \rho_1 \rho_{1S_1}} = \frac{\rho_2}{\rho_1} \frac{\Delta S_2^A - \frac{\rho_{1S_2}}{\rho_{1S_1}} \Delta S_1^A}{\Delta S_2^A - \frac{\rho_{2S_2}}{\rho_{2S_1}} \Delta S_1^A} > 1. \tag{33}$$

B Conditions for an inflection point. When non-monotony can be ruled out ($K < 1$), we are interested in capturing the possibility of a change of curvature of the accumulation path, characterized by a condition on the second-order

[9] which is what happens in case C; indeed we have :

$$S_1^C(t) = S_{1\infty}^C + C_1^C r_{11} e^{r1t} \Rightarrow \frac{dS_1^C(t)}{dt} = r_1 C_1^C r_{11} e^{r1t}, \text{ of fixed sign.}$$

[10] To rule out the trivial case of a nul initial stock.

time derivative of the carbon path :

$$\exists ?\ t^{S_1'} \text{ such that :} \frac{d^2 S_1^A(t)}{dt^2} = \sum_{i=1}^{2} \rho_i^2 C_i^A \rho_{iS_1} e^{\rho_i t} = 0.$$

The same method as above is used, so that :

$$t^{S_1'} = \frac{1}{\rho_1 - \rho_2} \ln \left[-\frac{C_2 \rho_2^2 \rho_{2S_1}}{C_1 \rho_1^2 \rho_{1S_1}} \right]. \tag{34}$$

The positivity of the term inside the ln is then guaranteed by the following condition : $K > \frac{\rho_1}{\rho_2}$. Adding the restriction that ensures monotony, we have shown point B of the Proposition.

Finally, when both non monotony and a change of curvature can be ruled out, point A applies.

Appendix 5 : Illustrative cases

Case 1. We are in the following set-up : $S_1^0 < S_{1\infty}^C < S_{1\infty}^A$. It is straightforward to show that $S_1^C(t)$ is monotically increasing and concave. Then, it suffices that $S_1^A(t)$ be increasing, initially convex then concave, to show that the sign of $S_1^A(t) - S_1^C(t)$ changes over time.

$S_1(t)^A$ is increasing means that $\forall t\ \frac{dS_1^A(t)}{dt} > 0$ (condition **C1**), in particular at $t = 0$. We want to check under which conditions this is compatible with an initially convex stock path, which is characterized by $\frac{d^2 S_1^A(t)}{dt^2}|_{t=0} > 0$ (condition **C2**).

$$\begin{aligned}
\mathbf{C1} &\Rightarrow C_1 \rho_1 \rho_{1S_1} + C_2 \rho_2 \rho_{2S_1} > 0 \\
&\Rightarrow C_1 \rho_1^2 \rho_{1S_1} + C_2 \rho_2^2 \rho_{2S_1} < 0 \text{ if, } \forall i, C_i \rho_i \rho_{iS_1} > 0 \text{ as } \rho_1, \rho_2 < 0 \\
&\Rightarrow \frac{d^2 S_1(t)}{dt^2}|_{t=0} < 0.
\end{aligned}$$

$$
\begin{aligned}
\mathbf{C1} &\Rightarrow C_2\rho_2\rho_{2S_1} > -C_1\rho_1\rho_{1S_1},\text{ if } C_1\rho_1\rho_{1S_1} < 0 \text{ and } C_2\rho_2\rho_{2S_1} > 0,\\
&\Rightarrow -\frac{C_2\rho_2\rho_{2S_1}}{C_1\rho_1\rho_{1S_1}} > 1\\
&\Rightarrow -\frac{C_2\rho_2^2\rho_{2S_1}}{C_1\rho_1^2\rho_{1S_1}} > \frac{\rho_2}{\rho_1} > 1\\
&\Rightarrow C_2\rho_2^2\rho_{2S_1} < -C_1\rho_1^2\rho_{1S_1}\\
&\Rightarrow \frac{d^2S_1(t)}{dt^2}|_{t=0} < 0.
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{C1} &\Rightarrow C_1\rho_1\rho_{1S_1} > -C_2\rho_2\rho_{2S_1},\text{ if } C_1\rho_1\rho_{1S_1} > 0 \text{ and } C_2\rho_2\rho_{2S_1} < 0,\\
&\Rightarrow 0 < -\frac{C_2\rho_2^2\rho_{2S_1}}{C_1\rho_1^2\rho_{1S_1}} < \frac{\rho_2}{\rho_1}\\
&\Rightarrow 0 < -\frac{C_2\rho_2^2\rho_{2S_1}}{C_1\rho_1^2\rho_{1S_1}} < 1 \text{ or } 1 < -\frac{C_2\rho_2^2\rho_{2S_1}}{C_1\rho_1^2\rho_{1S_1}} < \frac{\rho_2}{\rho_1}\\
&\Rightarrow \frac{d^2S_1(t)}{dt^2}|_{t=0} < 0 \text{ or } \frac{d^2S_1(t)}{dt^2}|_{t=0} > 0.
\end{aligned}
$$

Case 2. We are in the following set-up : $S_{1\infty}^C < S_1^0 < S_{1\infty}^A$. We are looking for conditions compatible with a non-monotonous optimal carbon accumulation, starting with a decreasing part. Furthermore, as the myopic path is non ambiguously decreasing, we also want to ensure that the slope of the tangent at $t = 0$ is higher in the myopic case than in the optimal case. This set of conditions straightforwardly leads to the one exposed in Proposition 2.

Case 3. We are in the following set-up : $S_{1\infty}^C < S_{1\infty}^A < S_1^0$. We are looking for conditions compatible with a non-monotonous optimal carbon accumulation, starting with a decreasing part. This set of conditions straightforwardly leads to the one exposed in Proposition 3.

Figures 4-6 are companion to Figures 1-3 as they illustrate sulphur accumulation and abatement over time.
[Figures 4,5,6 about here]

Appendix 6 : Different sets of values used in the numerical illustrations

Parameter	Figure 1	Figure 2	Figure 3
$\bar{e_1}$, $\bar{e_2}$	5, 5	10, 5	10, 5
δ_1, δ_2	0.1, 0.1	0.08, 0.1	0.08, 0.1
d_1, d_2	0.1 , 0.2	1, 1	1, 1
d_3, t_1, t_2	0.5, 1, 0.8	1, 1, 4	1, 1, 4
c_1, c_2	2, 3	2, 3	2, 3
ΔS_1^A	> 0	> 0	< 0
ΔS_1^C	> 0	< 0	< 0
ΔS_2^A	> 0	> 0	> 0
ΔS_2^C	> 0	< 0	> 0

Table 2: Parameters used in the simulations

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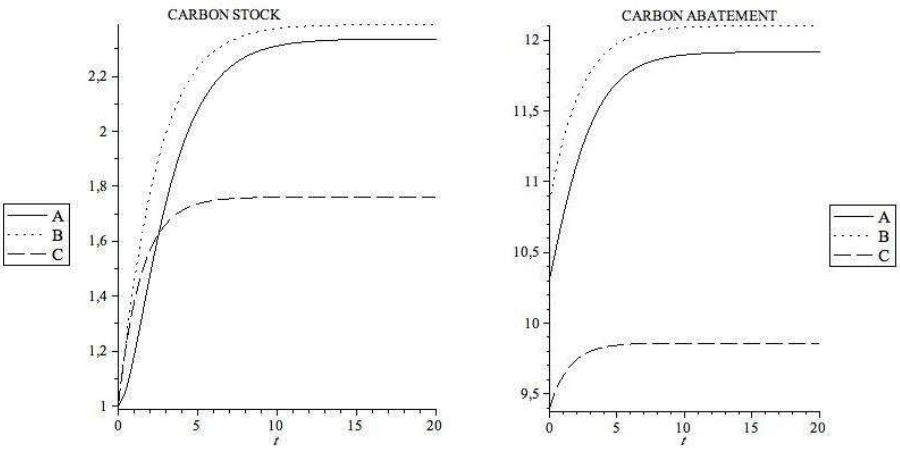

Figure 1: Illustration of Result 1

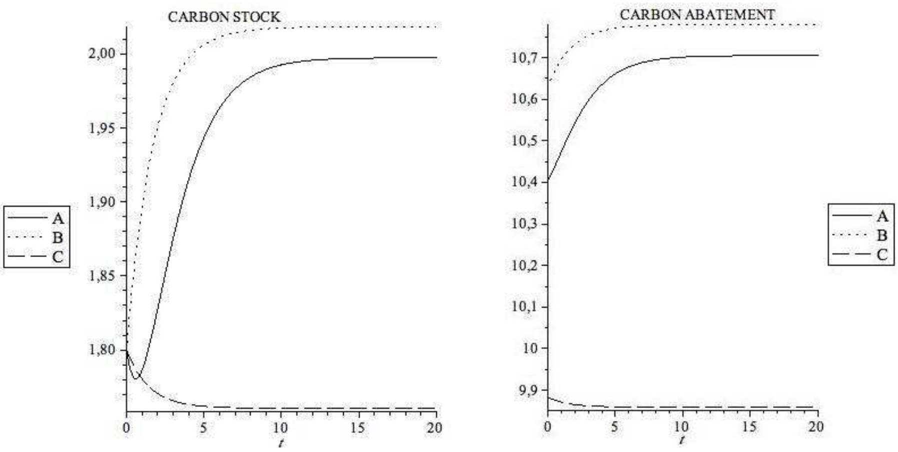

Figure 2: Illustration of Result 2

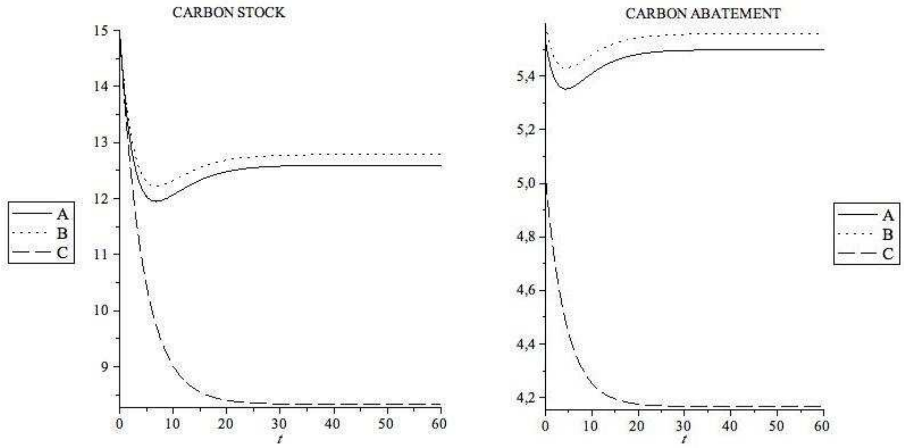

Figure 3: Illustration of Result 3

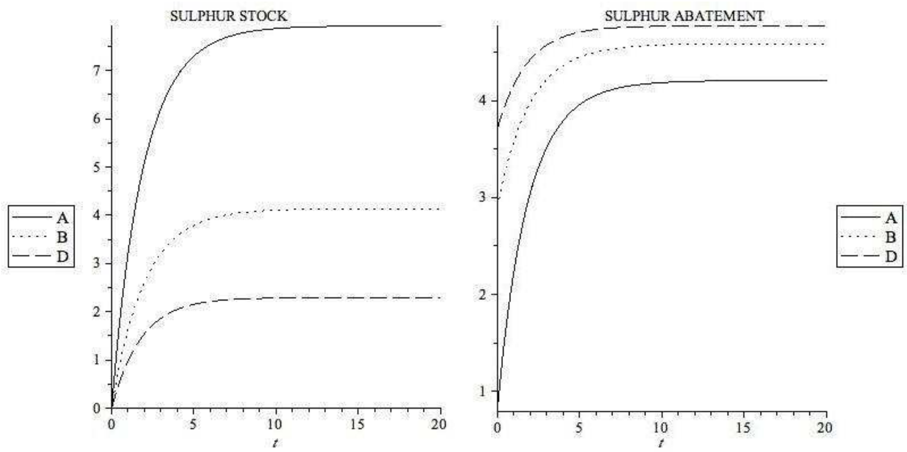

Figure 4: Complement to Figure 1

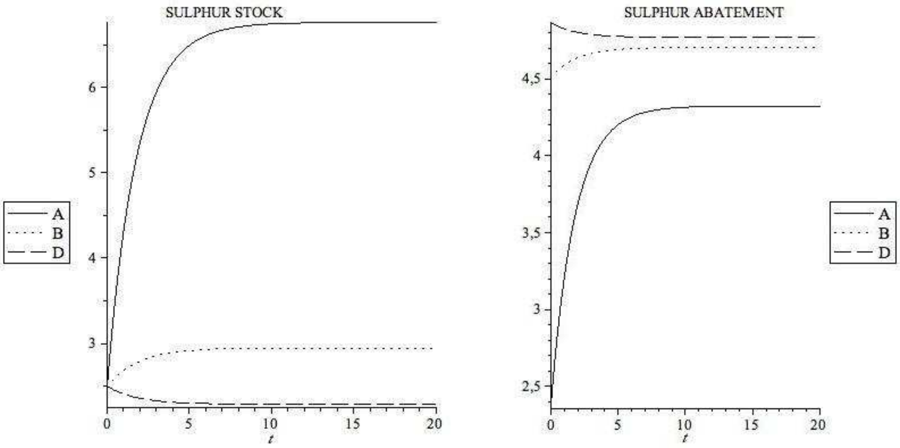

Figure 5: Complement to Figure 2

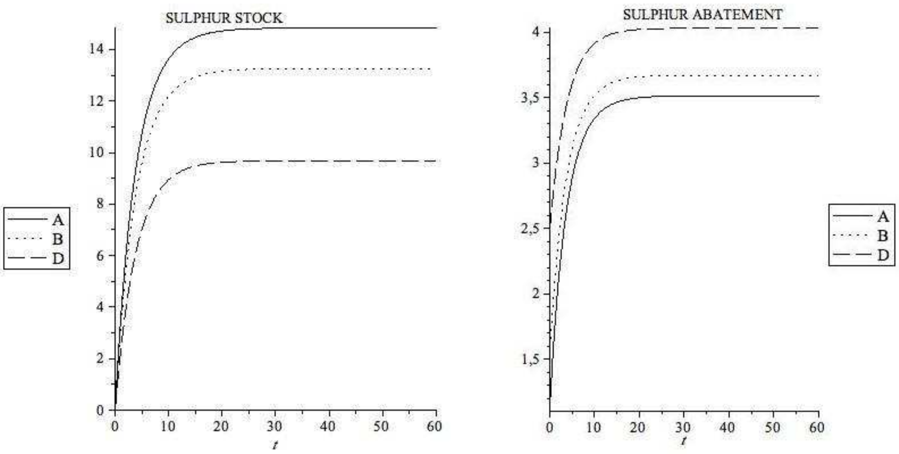

Figure 6: Complement to Figure 3