Agglomeration and Welfare: the core-periphery model in the light of Bentham, Kaldor, and Rawls*

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Abstract

The objective of this paper is to apply different welfare approaches to the canonical model developed by Krugman, with the aim of comparing the only two possible market outcomes, i.e. agglomeration and dispersion. More precisely, we use the Pareto criterion, the compensation criteria put forward by Kaldor, as well as the utilitarian and Rawlsian welfare functions. No clear answer emerges for the following two reasons: (i) except for small range of transport cost values, there is indetermination when compensation schemes are used and (ii) the best outcome heavily depends on societal values regarding inequalities across individuals. In particular, our analysis cautions against the use of utilitarian welfare functions as a foundation for regional policy recommendations.

Keywords: agglomeration, welfare, economic geography, compensation mechanism.

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1 Introduction

Recent developments in economic geography have focused on the reasons explaining the formation of economic agglomerations.[1] However, they have put aside the question of the social desirability of such agglomerations. Some authors have even expressed some reticence in doing such an analysis.[2] Hence, whether there is too much or too little agglomeration is still unclear, even in the canonical model developed by Krugman. Yet, speculation on this issue has never been in short supply and it is fair to say that this is one of the main questions that decision makers would like to address. There are several good reasons to believe that the market outcome is not efficient. Indeed, besides the standard inefficiencies generated by firms pricing above marginal costs, economic geography models contain new sources of inefficiency whose origin lays in the mobility of agents. Typically, firms and workers move without taking into account neither the benefits or losses they bring about to the agents residing in their new region, nor the benefits or losses they impose on those left behind. Accordingly, there is a priori no general indication as to the agglomeration or dispersion of the mobile factor is socially desirable.

The objective of this paper is to use Krugman's core-periphery model with the aim of identifying conditions under which agglomeration or dispersion is the better social outcome (hence, we do not conduct a first best analysis).[3] In order to compare the two market outcomes - agglomeration and dispersion - that most economic geography models yield, we will use the traditional tools of public economics, starting from the least controversial criterion to various social welfare functions. Let us make it clear that the answer to the question above is not an easy one. Even though the setting provided by Krugman involves no technological externalities at all, its welfare analysis does not deliver

[1]This has been accomplished by combining monopolistic competition, increasing returns at the plant level, mobile and immobile labor, and transport costs within a general equilibrium framework of the Dixit-Stiglitz genre. See Krugman (1991), Fujita *et al.* (1999), and Baldwin *et al.* (2003) for more details.

[2]As observed by Anas (2001), this is rather surprising because the Dixit-Stiglitz model of monopolistic competition, which is the main building-block of economic geography, was itself developed to deal with a welfare question: how does a market characterized by increasing returns and product differentiation perform from the social point of view?

[3]Other equilibria may exist but they are unstable.

a simple and unambiguous message. This is because the pecuniary externalities generated by the mobility of agents matter for welfare as the core-periphery model assumes monopolistic competition.

We show the following results. First, we will see that none of the two allocations Pareto dominates the other: the unskilled workers living in the periphery always prefer dispersion, whereas all those living in the core always prefer agglomeration. This is hardly a surprising result, given the terminology (core and periphery) used to describe the regions in the agglomerated configuration. We then turn to the *compensation mechanisms* put forward by Kaldor (1939) and Hicks (1940) to evaluate the social desirability of a move, using market prices and equilibrium wages to evaluate the compensations to be paid either by those who gain from the move (Kaldor), or by those who are hurt by the move (Hicks). This is not, however, the end of the story. As observed by Little (1949), it is not sufficient to look at the simple desirability of such compensations. We must also consider their feasibility at the corresponding equilibrium prices and wages. In other words, we must check that the compensations may be effectively paid and that the material balances conditions are met at the incomes earned by the workers after compensation. This can be achieved only within a general equilibrium framework, but this is precisely one of the main desirable features of the core-periphery model.

Given this proviso, we will show that, *provided that transport costs are sufficiently low, agglomeration is preferred to dispersion* in the following sense: all workers in the core can compensate those staying in the periphery, whereas those staying in the periphery are unable to compensate the workers who choose to move in what becomes the core. In addition, we do not just show the existence of transfers allowing for a Pareto-dominant move: we determine the precise value of these transfers. *Outside that range, we face a problem of indetermination* in the sense of Scitovsky (1941): none of the two configurations is preferred to the other with respect to the above two criteria. At first sight, this seems to be a surprising result. In our opinion, such an indetermination may be considered as the "synthesis" of the very contrasted views that prevail in a domain in which the two tenets have many good reasons to be right. To our knowledge, this is the first time that such a welfare analysis is performed in a spatial and general equilibrium context. We believe that this approach, which does not involve any interpersonal comparison and rests on market prices and incomes determined in a general equilibrium context, is superior to many others.

The partial indetermination mentioned in the foregoing leads us to focus on various social welfare functions. Even though utilitarianism may lead to awkward results in spatial models (Mirrlees, 1972; Wildasin, 1986), we follow a well-established tradition in public economics (see, e.g. Atkinson and Stiglitz, 1980) and consider the CES family of social welfare functions, which encapsulate different attitudes toward inequality across individuals and includes the utilitarian and Rawlsian criteria as polar cases. As expected, *the relative merits of agglomeration then critically depend on societal values.* If society does not care much about inequality across individuals, we show that agglomeration (resp., dispersion) is socially desirable once transport costs are below (resp., above) some threshold, the value of which depends on the fundamental parameters of the economy.[4] As the degree of aversion toward inequality rises, the domain of transport cost values for which agglomeration is preferred monotonically shrinks; it vanishes completely when the social welfare function is Rawlsian. Even though these results are derived from social preferences defined on individualistic utilities, they lead to policy recommendations that may be regarded as being region-based. This is due to the fact that, according to the core-periphery model, the market yields very contrasted distributions of skilled workers and income.

Somewhat surprisingly, we will see that the solution obtained by applying Kaldor's compensation principle is identical to that derived when the degree of aversion to inequality is equal to two. This result allows us to connect the main two approaches followed in our attempt to assess the relative merits of agglomeration and dispersion. It is also worth observing that the utilitarian criterion leads to prescriptions that differ in several important respects from those obtained by the welfare analysis based on compensations, showing once more how unsatisfactory the utilitarian approach may be. In particular, *the utilitarian criterion appears to be biased toward agglomeration in that it sustains this configuration for the largest range of transport cost values.*

[4]This is reminiscent of Ottaviano and Thisse (2002) who show that there is a whole range of transport costs for which agglomeration is the only stable equilibrium, whereas dispersion is socially desirable. Nevertheless, unlike Krugman's, their model does not allow for income effects. The similarity between the two results is therefore worth stressing.

The remainder of the paper is organized as follows. The model is presented in section 2. Using independent normalizations that entail no loss of generality, we establish several results leading to simple expressions, which are used in the welfare analysis of the agglomerated and dispersed configurations. In particular, we show that the nominal wage of the unskilled workers is identical to the nominal wage of the skilled workers, whatever their location and the equilibrium configuration. The Pareto, Kaldor and Hicks criteria are then studied in section 3, whereas the CES family of social welfare functions is used in section 4 to compare the two configurations. Section 5 concludes.

2 The model and some preliminary results

2.1 Assumptions and notation

Consider an economy with two regions, labelled 1 and 2, two sectors, the agricultural and the manufacturing sectors, and two types of labor, the skilled and the unskilled. There are L unskilled workers and H skilled workers. The spatial distribution of unskilled labor is fixed and uniform (that is, there are $L/2$ unskilled workers in each region), whereas the spatial allocation of skilled labor is endogenous. The agricultural sector produces a homogenous product under perfect competition and constant returns, using one unit of unskilled labor to produce one unit of output. This good is costlessly tradeable, thus implying that the wage of the unskilled workers is the same across regions. The agricultural good is chosen as the numéraire so that $p_a = w_a = 1$. The manufacturing sector produces a continuum of varieties of a horizontally differentiated good under monopolistic competition and increasing returns, using skilled labor as the only input. Any variety of this good can be shipped from one region to the other according to an iceberg technology described by the parameter $\tau > 1$, meaning that τ units must be sent from the region of origin for 1 unit to be available in the region of destination.

Preferences are identical across all workers. Each individual has Cobb-Douglas preferences over the traditional good and an aggregate of varieties of the manufacturing good, with a share of manufacturing expenditure μ and a constant elasticity of substitution across varieties $\sigma > 1$. Hence, tastes are described by the following utility function:

$$U = \left(\int_{i=0}^{N} q(i)^{(\sigma-1)/\sigma} \right)^{\frac{\sigma\mu}{\sigma-1}} z^{1-\mu} \tag{1}$$

where $q(i)$ and z respectively represent the quantity consumed of the manufacturing variety i and of the agricultural good by the representative individual. As a result, the demand for the agricultural good of a worker located in region $r = 1, 2$ is

$$z_r = (1 - \mu) w_r$$

irrespective of the region in which the good is produced since trading the agricultural product is costless.

By the same token, the demand q_{jr} for a variety produced in region j of a worker located in region $r = 1, 2$ is as follows:

$$q_{jr} = \frac{p_{jr}^{-\sigma}}{P_r^{1-\sigma}} \mu w_r$$

where p_{jr} is the common consumer price - i.e. inclusive of transport costs - of a variety produced in j and sold in r, w_r the nominal wage prevailing in region r, and P_r the price index of the aggregate of varieties in region r, defined by

$$P_r = \left[n_r p_r^{1-\sigma} + n_s (\tau p_s)^{1-\sigma} \right]^{1/(1-\sigma)} \qquad r = 1, 2 \text{ and } s \neq r \tag{2}$$

with n_r (resp., n_s) being the number of varieties produced in region r (resp., $s \neq r$) and p_r (resp., p_s) the common producer price - i.e. net of transport costs - of the varieties produced in region r (resp., $s \neq r$). Because demand is iso-elastic, mill-pricing is an equilibrium feature of the model. Formally, this can be seen in (2), where transportation costs are fully passed onto consumers (indeed, τp_s is the consumer price for region r consumers).

Consequently, the market demand for variety i produced in region r is given by

$$D_r(i) = \mu (P_r^{\sigma-1} Y_r + \tau^{1-\sigma} P_s^{\sigma-1} Y_s) p_r^{-\sigma} \tag{3}$$

where Y_r is the income of region r. Since there is free entry and exit, therefore zero profits in equilibrium, the income in region 1 (resp., 2) is given by:

$$Y_1 = \frac{L}{2} + w_1 \lambda H \qquad Y_2 = \frac{L}{2} + w_2 (1 - \lambda) H$$

where λ is the endogenous share of skilled workers located in region 1.

The distribution of skilled workers λ is governed by the difference of individual welfare between the two regions. Recalling that $p_a = 1$ by our choice of numéraire, the welfare of a worker in region r is evaluated by her indirect utility function given (up to a multiplicative constant) by

$$V_r = w_r P_r^{-\mu} \qquad r = 1, 2 \tag{4}$$

Hence, the mobility of the skilled (and of firms) is determined by the real wage differential. All things being equal, the price index is lower in the region with the larger number of firms, thus implying that the skilled workers are attracted by places where firms are many (see (2)). This agglomeration force is called the *price index effect*.

Turning to the supply side in region r, the production of the quantity q_r of any variety requires h_r units of skilled labor given by:

$$h_r = \alpha + \beta q_r \qquad r = 1, 2 \tag{5}$$

where $\alpha > 0$ and $\beta > 0$ are respectively the fixed and the marginal labor requirements. Hence, production of any variety exhibits increasing returns to scale internal to the firm. The profit function of each firm located in region r is therefore $\pi_r = p_r q_r - w_r(\alpha + \beta q_r)$.

Without loss of generality, we may choose the unit of the manufactured good such that $\alpha = 1/\sigma$. Following Baldwin *et al.* (2003), we may also use the fact that the number of firms is continuous and choose the unit of the real line along which this number is measured in order to have $\beta = (\sigma - 1)/\sigma < 1$.[5] The equilibrium price p_r, which maximizes profits under demand constraint is then obtained by applying the first order condition:[6]

$$p_r = w_r \qquad r = 1, 2 \tag{6}$$

whereas the zero-profit condition implies that the equilibrium output of any active firm is

$$q_r = 1 \qquad r = 1, 2 \tag{7}$$

[5]Doing so reduces the proliferation of parameters in the model. See Neary (2001) and Baldwin *et al.* (2003) for a discussion.

[6]Note that (6) implies markup pricing since the marginal cost is equal to $\beta w_r < w_r$ by our normalisation of β, which is smaller than 1.

Nominal wages adjust so that no firms want to enter or exit the market, that is, profits are zero. Using (3), (2) and (6), we find that the typical firm established in region r breaks even if

$$w_r = w_r^{1-\sigma} \frac{\mu Y_r}{n_r w_r^{1-\sigma} + \tau^{1-\sigma} n_s w_s^{1-\sigma}} + \tau^{1-\sigma} w_r^{1-\sigma} \frac{\mu Y_s}{\tau^{1-\sigma} n_r w_r^{1-\sigma} + n_s w_s^{1-\sigma}}$$

Obviously, this expression is nonlinear in w_r, which explains why most of the economic geography literature relies on simulations for most of their results. As we shall see, we can nevertheless characterize most interesting aspects of the model with pencil and paper.

We also need an expression for the unskilled labor as well as an expression for the agricultural good markets clearing conditions. By our assumptions and normalizations about the agricultural sector, these are the same. In particular, they are given by

$$L = (1 - \mu)(w_1 H_1 + w_2 H_2 + L) \tag{8}$$

Inter-regional trade balances regional excesses in supply and demand across sectors.

Finally, the skilled labor supply in regions 1 and 2 is given by λH and $(1 - \lambda)H$, respectively. The skilled labor demand from a typical firm located in region r is given by (5). Hence, given the normalization above, each skilled labor market clearing implies that the number of firms located in region 1 and 2 as well as the total mass of firms, are such that:

$$n_1 = \lambda H \qquad n_2 = (1 - \lambda)H \qquad n = n_1 + n_2 = H$$

Thus, the total number of firms (n) is constant but their spatial distribution is endogenous because skilled workers are mobile. In addition, there is a one-to-one relationship between workers and firms established in the same region which, therefore, move together.

Firms are attracted by the region where workers are numerous because demand for the manufactured good is high (see (3)). This agglomeration force is called the *market size effect*, which strengthens the price index effect to generate a circular causation, driving the agglomeration of firms and skilled workers into one region. By contrast, the immobility of the unskilled is a dispersion force.

A spatial equilibrium is the outcome of the interplay between those agglomeration and dispersion forces and is such that no skilled worker has an incentive to change location. It depends on the transport cost level (Krugman, 1991). Formally, denoting by $V_i(\lambda)$ the indirect utility a skilled worker enjoys in region $i = 1, 2$, a *spatial equilibrium* arises at $\lambda \in (0, 1)$ when

$$\Delta V(\lambda) \equiv V_1(\lambda) - V_2(\lambda) = 0$$

or at $\lambda = 0$ when $\Delta V(0) \leq 0$, or at $\lambda = 1$ when $\Delta V(1) \geq 0$. Such an equilibrium always exists because $V_r(\lambda)$ is a continuous function of λ (Ginsburgh *et al.*, 1985). A spatial equilibrium is (locally) *stable* if, for any marginal deviation of the population distribution from the equilibrium, the equation of motion

$$\dot{\lambda} = \lambda \Delta V(\lambda)(1 - \lambda)$$

brings the distribution of skilled workers back to the original one.

2.2 Properties of the spatial equilibrium

It is well known that the model presented in the foregoing section leads to very contrasted spatial equilibria according to the level of transport costs. We summarize below the main conclusions derived so far from the core-periphery model; details of the proofs may be found in Fujita *et al.* (1999), Fujita and Thisse (2002), Neary (2001) and Baldwin *et al.* (2003).[7] Agglomeration, namely $\lambda = 0, 1$, is the unique stable equilibrium, whatever the transport cost level, when varieties are very differentiated, that is, when $\sigma < \overline{\sigma}$ (which is called the black hole condition) with

$$\overline{\sigma} \equiv \frac{1}{1 - \mu} \tag{9}$$

When the no black hole condition holds ($\sigma \geq \overline{\sigma}$), agglomeration is a stable equilibrium if $\tau \in [1, \tau_s]$ where τ_s is a the unique solution in the interval $(1, \infty)$ of the equation:

$$\frac{1 - \mu}{2} \tau^{\sigma(1-\mu)-1} + \frac{1 + \mu}{2} \tau^{1-\sigma(1+\mu)} = 1$$

[7] A very comprehensive study of the set of equilibria and of their stability properties in this and related models can be found in Robert-Nicoud (2002).

whereas dispersion, namely $\lambda = 1/2$, is a stable equilibrium, if $\tau \in (\tau_b, \infty)$ where

$$\tau_b = \left\{ \frac{(1+\mu)[\sigma(1+\mu)-1]}{(1-\mu)[\sigma(1-\mu)-1]} \right\}^{1/(\sigma-1)}$$

Furthermore, we have

$$\tau_s > \tau_b$$

so that both dispersion and agglomeration are stable equilibria when transport costs lie within the interval $[\tau_b, \tau_s]$. When τ lies in the interval (τ_b, τ_s), there also exist two interior, asymmetric equilibria, symmetric around $\lambda = 1/2$, but these are unstable.

Let us now show that, at any stable spatial equilibrium, prices as well as nominal wages are equal and constant for all skilled workers regardless of the type of equilibrium - agglomeration (A) or dispersion (D). When the economy involves dispersion, we have $H_1 = H_2 = H/2$ and $w_1^D = w_2^D = w^D$ so that (8) implies

$$w^D = \frac{\mu}{1-\mu}\frac{L}{H}$$

When the economy involves agglomeration (in region 1, say), we have $H_1 = H$, $H_2 = 0$, $w_1^A = w^A$ so that (8) leads to

$$w^A = \frac{\mu}{1-\mu}\frac{L}{H}$$

which is equal to w^D. We have shown:

Lemma 1 *Nominal wages of the skilled workers are identical, whether the economy is dispersed or agglomerated.*

In the sequel, we use two additional normalizations in order to simplify the expression of w^D and w^A. More precisely, we choose the unit of skilled labor and the unit of unskilled labor in the economy to satisfy the equalities $H = \mu$ and $L = 1 - \mu$, from which it follows immediately that

$$w_r = w^D = w^A = 1 \tag{10}$$

Since $w_a = 1$, we have:

Lemma 2 *If the unit of skilled (reps., unskilled) labor is chosen for the proportion of skilled (resp., unskilled) workers to be equal to the share of expenditure* *on the manufactured (resp., agricultural) good, whatever the market equilibrium, the nominal wages per unit of skilled and unskilled labor are identical for both types of workers.*

Hence, real wages vary between agglomeration and dispersion only with the price index.[8] Using (6) then shows that *the price of local varieties is the same* (and equal to 1) *in both regions at either equilibrium.* Consequently, (10) implies that the indirect utility varies only with the spatial distribution of firms through the price index. Indeed, substituting (2) into (4), we obtain:

$$V_r = (n_r + \tau^{1-\sigma} n_s)^{\mu/(\sigma-1)} \tag{11}$$

Note that this expression is valid only for $n_r \in \{0, H/2, H\}$, for otherwise w_r and p_r are different from unity.

When there is dispersion, let q_{rr}^D (resp., q_{rs}^D) be the equilibrium consumption of a worker located in region r of any variety produced in region r (resp., region s). The expressions of q_{rs}^D and q_{rr}^D are respectively given by

$$\begin{aligned} q_{rs}^D &= \mu P_r^{\sigma-1} \tau^{-\sigma} = \frac{2\tau^{-\sigma}}{1+\tau^{1-\sigma}} \\ q_{rr}^D &= \mu P_r^{\sigma-1} = \frac{2}{1+\tau^{1-\sigma}} \end{aligned} \tag{12}$$

thus implying that

$$q_{rr}^D = \tau^{\sigma} q_{rs}^D \tag{13}$$

Note that, when dispersion prevails, the consumption of a local (or of an imported) variety is the same whatever the location and the type of worker.

When there is agglomeration, the equilibrium consumptions of a worker located in region 1 (the core) and in region 2 (the periphery) are the same within each region but differ across regions. They are now given by:

$$\begin{aligned} q_{11}^A &= \mu P_1^{\sigma-1} = 1 \\ q_{21}^A &= \tau^{-1} q_{11}^A = \tau^{-1} \end{aligned}$$

Since $\tau > 1$ and $\sigma > 1$, we have the following ranking of those various consumption levels:

$$q_{rr}^D > q_{11}^A > q_{21}^A > q_{rs}^D$$

[8]Note that Lemma 2 does not mean that the number of skilled labor units embodied in a skilled worker is equal to the number of unskilled labor units embodied in a unskilled

The following comments are in order. First, under agglomeration, because of transport costs, workers living in the core consume more of each variety than those living in the periphery ($q_{11}^A > q_{21}^A$). Second, under dispersion, the presence of transport costs leads workers to consume more of each of the locally produced varieties and less of each of those produced abroad ($q_{rr}^D > q_{rs}^D$). The last two comparisons are less straightforward. Because of transport costs, workers substitute local varieties to imported varieties and end up consuming more of each of the locally produced varieties under dispersion than under agglomeration in which they equally consume each variety ($q_{rr}^D > q_{11}^A$). Similarly, although no variety is locally produced in the periphery, workers living in the periphery consume more of each imported variety under agglomeration than they do under dispersion ($q_{21}^A > q_{rs}^D$), because there is no substitution effect within the range of varieties.

We now have everything at hand to address the core issue of this paper.

3 Welfare analysis of the market equilibrium

3.1 Does agglomeration Pareto dominate dispersion, or vice versa?

When agglomeration arises, recall that it does so in region 1 by convention. The welfare of a skilled worker when there is agglomeration in region 1 is denoted $V_{H,1}^A$. When there is dispersion the welfare level is the same for all workers, $V_1^D = V_2^D = V^D$, because the number of firms located in each region is the same ($n_1 = n_2 = \mu/2$). Using (11), it is readily verified that $V_{H,1}^A$ and V^D are as follows:

$$V_{H,1}^A = 1 \qquad V^D = \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu}{\sigma-1}} \tag{14}$$

We now determine when $V_{H,1}^A > V^D$. This holds true if and only if $1 > [(1+\tau^{1-\sigma})/2]^{\frac{\mu}{\sigma-1}}$, that is, if and only if $1 > \tau^{1-\sigma}$, which is always satisfied because $\tau > 1$ (and $\sigma > 1$). Thus, we have the following result:

Proposition 3 *Whatever the level of transport costs, all skilled workers prefer agglomeration to dispersion.*

Consider the welfare of the unskilled in each equilibrium. The welfare of an unskilled worker located in the core is the same as the one of a skilled worker. Therefore, her well-being increases when agglomeration arises. By contrast, an unskilled worker living in the periphery is worse off. Indeed, such a worker prefers agglomeration to dispersion when $V^A_{L,2} \geq V^D$ with:

$$V^A_{L,2} = \tau^{-\mu} \tag{15}$$

Using (14), we can see that $V^A_{L,2} \geq V^D$ if and only if $\tau^{-\mu} \geq [(1+\tau^{1-\sigma})/2]^{\frac{\mu}{\sigma-1}}$, that is, $\tau < 1$, which is impossible. Thus, we have:

Proposition 4 *Whatever the level of transport costs, all the unskilled located in the periphery prefer dispersion to agglomeration whereas all the unskilled located in the core prefer agglomeration to dispersion.*

It follows from Propositions 3 and 4 that *neither agglomeration nor dispersion is a Pareto dominant allocation*: $V^A_{H,1} > V^D > V^A_{L,2}$. This result suffices to show that there is a conflict of interest between the two geographical groups of unskilled. The agglomeration of firms and skilled workers in one region implies a fall in the price index in this region, but leads to an increase in the price index in the other region because the unskilled living there now have to bear the transport costs of *all* varieties of the manufactured good. Hence, when skilled workers move from one region to the other, they impose a positive external effect on the immobile workers located in the core *but* a negative external effect on the immobile workers located in the periphery.

3.2 Agglomeration, dispersion and potential Pareto improvements

Without carrying out interpersonal comparisons, a compensation criterion based on the prevailing equilibrium prices and wages may be used in order to determine whether the level of social welfare rises when agglomeration or dispersion prevails. Restricting ourselves to these two configurations is legitimate because they are the only market equilibria. Let us describe how the compensation mechanism works.

Consider the following two states for the economy: agglomeration (A) and dispersion (D). Assume that agglomeration prevails, so that the workers living in the core (resp., periphery) are better off (resp., worse off) than under dispersion. We then ask the question: is A preferred to D? Two cases must then be distinguished. In the former, we follow Kaldor (1939) and say that A is preferred to D when the winners are able to compensate the losers in order to give them the utility level they would reach under dispersion. In the latter, we follow Hicks (1940) and say that A is preferred to D when the losers are not able to compensate the winners by giving them the utility level they would reach under agglomeration, when the economy moves to dispersion. As argued by Scitovsky (1941), both criteria must be satisfied for A to be preferred to D (or vice versa).

In either case, we take as given the equilibrium prices of the varieties as well as the equilibrium wages to determine the compensations to be paid. Consequently, for the compensations to be feasible, it must be that the individual consumptions of each variety evaluated at the *incomes net of compensation* add up to the quantity supplied by each firm. In this case, the material balance conditions hold true so that the equilibrium prices remain the same. These conditions must also be satisfied for the firms to be able to pay the equilibrium wages on which the compensations are based (see (10)).

Consider now the above two criteria.

1. *When does agglomeration* ($\lambda = 1$) *correspond to a potential Pareto improvement compared to dispersion* ($\lambda = 1/2$)? More precisely, when agglomeration prevails, we must determine if an appropriate redistribution of income from those workers residing in the core - who are better off - can keep unchanged the utility level of the unskilled residing in periphery - who are worse off.

The argument involves three steps: (i) we compute the transfers needed to compensate those who are in the periphery, (ii) we then check their feasibility and (iii) finally, we determine under which condition (if any) these transfers are desirable from the standpoints of the workers in the core.[9]

(i) For the unskilled in the periphery to be exactly compensated, they must be given an additional income C_A such that

$$\left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu}{\sigma-1}} = \tau^{-\mu}(1+C_A)$$

[9] From (10), it is clear that prices are wages are the same at any (stable) spatial equilibrium. However, prices and wages differ out of these equilibria. As a consequence, the compensation under consideration cannot be considered as non-distortionary in general.

whose value is equal to

$$C_A = \left(\frac{1+\tau^{1-\sigma}}{2\tau^{1-\sigma}} \right)^{\frac{\mu}{\sigma-1}} - 1 \tag{16}$$

which is positive because $\tau > 1$.

Since the residents in the periphery have the same welfare level and face the same prices and wage, the total compensation paid by those living in the core must be equal to

$$\frac{1-\mu}{2} C_A$$

Since the residents in the core have the same welfare level and face the same prices and wage, each resident in the core must then pay the same amount $T_A > 0$ given by

$$T_A = \frac{1-\mu}{1+\mu} C_A \tag{17}$$

which decreases as the size of the manufactured sector rises. After compensation, the income of a worker in the core is $1 - T_A$.

(ii) We must now determine if the material balance conditions still hold at the consumption pattern corresponding to the compensated incomes given by

$$Y_1 = \frac{1+\mu}{2}(1 - T_A) \qquad Y_2 = \frac{1-\mu}{2}(1 + C_A)$$

At the agglomerated configuration, the total consumption of every variety *after* compensation is given by the sum of the consumption by each of the workers who are in the core augmented by the consumption of each of the unskilled who are in the periphery. It follows from (3) and (8) that the total demand for any variety i - which are here all produced in region 1 - is such that

$$D_1(i) = \mu(P_1^{\sigma-1} Y_1 + \tau^{1-\sigma} P_2^{\sigma-1} Y_2)$$

where, given (2)

$$P_1 = \mu^{\frac{1}{1-\sigma}} \qquad P_2 = \tau \mu^{\frac{1}{1-\sigma}}$$

Replacing in $D_1(i)$ yields

$$D_1(i) = \mu \left[\frac{1}{\mu} \frac{1+\mu}{2}(1 - T_A) + \tau^{1-\sigma} \frac{\tau^{\sigma-1}}{\mu} \frac{1-\mu}{2}(1 + C_A) \right] = 1$$

with the second equality stemming from (16) and (17); this is just equal to the equilibrium production of a firm (see (7)). This production level allows each of them to pay the equilibrium wage ($w_r = 1$) used to calculate the compensation.

It follows from the Walras law that the market clearing condition for the agricultural good holds.

(iii) Finally, every worker in the core strictly prefers the agglomeration outcome if and only if her welfare level after compensation exceeds the welfare level she would get under dispersion, namely

$$1 - T_A > \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu}{\sigma-1}}$$

Using (16) and (17), this is equivalent to

$$F(\tau) \equiv \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{-\mu}{\sigma-1}} - \left(\frac{1+\mu}{2} + \frac{1-\mu}{2}\tau^{\mu}\right) > 0 \qquad (18)$$

As shown in the appendix, there exists a single value of $\tau > 1$, denoted τ_K, such that $F(\tau) = 0$ and $F(\tau) > 0$ if and only if $\tau < \tau_K$.

Consequently, when the economy moves from dispersion to agglomeration, *A is preferred to D in the sense of Kaldor as long as $\tau < \tau_K$ because the welfare benefits earned by those living in the core are sufficiently large to compensate those who stay in the periphery.* By contrast, when $\tau \geq \tau_K$, D is preferred to A.

2. *When does dispersion ($\lambda = 1/2$) correspond to a potential Pareto improvement compared to agglomeration ($\lambda = 1$)?* In other words, we want to know if, under dispersion, an appropriate redistribution of income from the unskilled who would otherwise be living in the periphery can keep unchanged the utility level of those who would live in the core, without making those unskilled worse off.

The answer is here straightforward. The payment of any compensation makes the spatial distribution of income uneven between regions. This in turn prevents the wages and prices corresponding to the dispersed configuration to balance the product and labor markets. Consequently, when the economy moves from dispersion to agglomeration, *the unskilled in region 2 are unable to compensate the other workers at the prevailing market prices and wages for the economy to move (back) toward dispersion.* Hence, A is always preferred to D in the sense of Hicks.

Accordingly, when $\tau < \tau_K$ agglomeration is preferred to dispersion according to both criteria.[10] By contrast, when $\tau \geq \tau_K$, we are in a situation of

[10] Note, however, the political difficulty of implementing the compensation to be paid by

indetermination in the sense of Scitovsky (1941), which means that no state of the economy is preferred under the two compensation criteria. We may then conclude as follows:

Proposition 5 *When transport costs are sufficiently low, agglomeration is socially preferable to dispersion in the sense of both Kaldor's and Hicks' criteria. Otherwise, it is impossible to discriminate between these two configurations by using those criteria.*

In order to assess the practical relevance of this result, we compute τ_K for different values of σ and μ chosen according to the following criteria. When the industrial sector stands for all tradeable goods in the economy, estimations of σ vary between 3 and 5 whereas μ takes values between 0.5 and 0.8 (Head and Mayer, 2003). By contrast, when this sector is a specific industry, σ sharply rises because varieties are now much better substitutes than in the aggregate level; a value of σ close to 7 is then acceptable. In this case, μ typically takes a value lower than 0.2, which corresponds approximately to the share of the manufactured good in a developed economy. Table 1, where the value between parentheses is the corresponding break point, reveals several interesting things.

First, the threshold τ_K seems to be lower than the break point τ_b so that the market would yield agglomeration for values of the transport costs that exceed the threshold below which it is socially desirable. When there is a black hole, i.e. $\sigma \leq 1/(1-\mu)$, agglomeration is always the market equilibrium, which means that $\tau_b \to \infty$ and $\tau_s \to \infty$. Because τ_K is strictly larger than 1 and takes a finite value, it must be that $\tau_K < \tau_b, \tau_s$. By continuity, we get $\tau_K < \tau_b$ for values of σ and μ that are just away from the domain defined by the black hole condition. However, when we move to parameter configurations that are sufficienly far from this domain (whence, agglomeration forces are weak), we obtain $\tau_b < \tau_K$. For example, for $\mu = 0.6$, we have $\tau_b < \tau_K$ as long as σ exceeds 12. This implies that *no general conclusion may be drawn in the Krugman core-periphery model as to whether the market yields excessive*

the workers in the core: the majority of workers $(1+\mu)/2$ is located in the core against a minority $(1-\mu)/2$ in the periphery. Evidence related to such political economy effects may be found in several European countries such as Belgium, Italy and Spain. See Robert-Nicoud and Sbergami (2003) for a possible theoretical justification of these effects.

agglomeration. Yet, for resonable values of the parameters μ and σ, excessive agglomeration seems to occur.

Second, for relatively low values of μ, the threshold τ_K is lower than the estimate of the transport cost τ obtained by Head and Ries (2001) from data relative to Canada-US trade, once they have controlled for other spatial differences, thus suggesting that τ_K may take values that are plausible. Last, the gap between τ_b and τ_K sharply rises when the economy moves from the lower left corner of the table toward the upper right corner, namely when we consider a larger industrial sector producing more differentiated varieties. In this case, the market outcome seems to be inefficient over a expanding range of transport cost values.

	$\mu = 0.2$	$\mu = 0.4$	$\mu = 0.6$
$\sigma = 3$	1.45 (1.67)	2.07 (3.05)	3.11 (8.72)
$\sigma = 5$	1.22 (1.26)	1.49 (1.63)	1.94 (2.30)
$\sigma = 7$	1.14 (1.16)	1.32 (1.36)	1.60 (1.68)

Table 1. Critical values of τ for the compensation mechanisms

4 Market equilibrium and social welfare functions

4.1 Agglomeration or dispersion: the impact of societal values

The partial indetermination established in Proposition 5 leads us to retain particular, but meaningful, social welfare functions (SWF), which are often used in public economics. Although the SWF approach is debatable, we believe that it contributes to fill up the gap (at least partially) in the welfare analysis of the core-periphery model.

In the case of n agents whose utility is $u_i(s)$ when the state of the economy is s, we focus on the following class of symmetric CES-type SWF:

$$W(s) = \begin{cases} \frac{1}{1-\eta} \sum_{i=1}^{n} [u_i(s)]^{1-\eta} & \text{for } \eta \neq 1 \\ \\ \sum_{i=1}^{n} \ln u_i(s) & \text{for } \eta = 1 \end{cases} \tag{19}$$

in which $\eta \geq 0$ measures the degree of aversion toward inequality. In particular, when $\eta = 0$ (or zero aversion to inequality) the function W is identical to the *utilitarian* welfare function in which the sum of all workers' (indirect) utilities is maximized.[11] At the other extreme, when $\eta \to \infty$ (or infinite aversion to inequality), we have the *Rawlsian* welfare function in which the (indirect) utility of the worst-off worker is maximized. Intermediate values of η express different societal attitudes toward economic inequality among individuals, that is, among three groups of workers in our setting: as η rises from zero to infinity, the bias in favor of the disadvantaged increases.

As in the foregoing, we want to compare two states of the economy: agglomeration and dispersion. In the first one, our SWF takes the values

$$W(A) = \frac{1}{1-\eta}\left[\frac{1+\mu}{2}(V_{H,1}^A)^{1-\eta} + \frac{1-\mu}{2}(V_{L,2}^A)^{1-\eta}\right]$$

where $V_{H,1}^A$ and $V_{L,2}^A$ are given in (14) and (15) and where we have accounted for the fact that the skilled and unskilled workers living in the core have the same utility level. When dispersion prevails, we have:

$$W(D) = \frac{1}{1-\eta}\left(V^D\right)^{1-\eta}$$

where V^D is the common utility level defined in (14). Using (14) and (15), we see that yields

$$G(\tau;\eta) \equiv W(A) - W(D) = \frac{1}{1-\eta}\Delta(\tau;\eta)$$

where

$$\Delta(\tau;\eta) \equiv \left(\frac{1+\mu}{2} + \frac{1-\mu}{2}\tau^{-\mu(1-\eta)}\right) - \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu(1-\eta)}{\sigma-1}}$$

Clearly, we have $G(1;\eta) = \Delta(1;\eta) = 0$. To determine the sign of $G(\tau;\eta)$ for $\tau > 1$, it is useful to compute its derivative, hence the derivative of $\Delta(\tau;\eta)$, with respect to τ:

$$\frac{d\Delta}{d\tau} = \mu(1-\eta)\left[-\frac{1-\mu}{2}\tau^{-\mu(1-\eta)-1} + \frac{\tau^{-\sigma}}{2}\left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu(1-\eta)}{\sigma-1}-1}\right]$$

[11] Even though individual utilities have the Gorman form, the utilitarian approach is not easy to justify here, for workers living in different regions face different ranges of prices because the number of imported varieties changes with the location of firms.

The sign $d\Delta/d\tau$ changes when η crosses the value 1. However, the sign of the derivative of $G(\tau;\eta)$ with respect to τ is unaffected because $\Delta(\tau;\eta)$ is itself divided by $1-\eta$ in $G(\tau;\eta)$.

The equation $d\Delta/d\tau = 0$ has a single root given by

$$\tau' \equiv \left[2(1-\mu)^{\frac{1-\sigma}{\sigma-\mu(1-\eta)-1}} - 1\right]^{1/(\sigma-1)}$$

This root exceeds 1 if and only if

$$\sigma > 1 + \mu(1-\eta)$$

which holds as long as the no black hole condition is satisfied. Moreover, we have

$$\lim_{\tau\to\infty} \Delta(\tau;\eta) = \frac{1+\mu}{2} - \left(\frac{1}{2}\right)^{\frac{\mu(1-\eta)}{\sigma-1}}$$

This limit is positive if and only if $\sigma < \widehat{\sigma}(\eta)$, where

$$\widehat{\sigma}(\eta) \equiv 1 + (1-\eta)\frac{\mu \log 2}{\log 2 - \log(1+\mu)}$$

Therefore, as long as $\sigma \leq \widehat{\sigma}(\eta)$, $G(\tau;\eta)$ is (weakly) positive for all $\tau > 1$ (see Figure 1a). It is clear that the threshold $\widehat{\sigma}(\eta)$ exceeds 1 if and only if $\eta < 1$. As a result, when varieties are sufficiently differentiated for σ to be very small and when the degree of aversion toward inequality is weak enough, agglomeration is always preferred to dispersion.

Insert Figure 1 about here

It remains to consider the case where $\sigma \geq \widehat{\sigma}(\eta)$.[12] Then, we know that $\tau' > 1$. It is readily verified that $dG/d\tau$ is positive for $\tau < \tau'$ and negative for $\tau > \tau'$, regardless of the value of η. This implies the existence of a unique value $\tau^*(\eta) > \tau' > 1$ such that $G[\tau^*(\eta);\eta] = 0$ (see Figure 1b). Accordingly, as long as τ is lower than this threshold, agglomeration is preferred to dispersion whereas the opposite holds when τ is larger than $\tau^*(\eta)$.

The foregoing results may then be summarized as follows:

[12]Note that this condition is always satisfied for $\eta \geq 1$.

Proposition 6 *When varieties are very differentiated ($\sigma \leq \widehat{\sigma}(\eta)$), agglomeration is preferred to dispersion, whatever the level of transport costs. When they are not ($\sigma > \widehat{\sigma}(\eta)$), there exists a transport cost value $\tau^*(\eta) > 1$ such that agglomeration (resp., dispersion) is preferred to dispersion (resp., agglomeration) when $\tau < \tau^*(\eta)$ (resp. $\tau > \tau^*(\eta)$).*

This result leads us naturally to investigate how $\tau^*(\eta)$ varies with η. By definition of $\tau^*(\eta)$, we have

$$\frac{1}{1-\eta}\left[\frac{1+\mu}{2}+\frac{1-\mu}{2}(\tau^*(\eta))^{-\mu(1-\eta)}\right]=\frac{1}{1-\eta}\left(\frac{1+(\tau^*(\eta))^{1-\sigma}}{2}\right)^{\frac{-\mu(1-\eta)}{\sigma-1}}$$

which is equivalent to

$$\left[\frac{1+\mu}{2}+\frac{1-\mu}{2}(\tau^*(\eta))^{-\mu(1-\eta)}\right]^{1/(1-\eta)}=\left(\frac{1+(\tau^*(\eta))^{1-\sigma}}{2}\right)^{\frac{-\mu}{\sigma-1}}$$

Since the LHS of this expression is a decreasing function of η (Avriel, 1972), for $\widehat{\eta} < \eta$ it must be that

$$\left[\frac{1+\mu}{2}+\frac{1-\mu}{2}(\tau^*(\eta))^{-\mu(1-\widehat{\eta})}\right]^{1/(1-\widehat{\eta})}>\left(\frac{1+(\tau^*(\eta))^{1-\sigma}}{2}\right)^{\frac{-\mu}{\sigma-1}} \qquad (20)$$

Since the LHS of (20) decreases whereas its RHS increases when $\tau^*(\eta)$ rises, the equality between the two sides of (20) is reached at $\tau^*(\widehat{\eta}) < \tau^*(\eta)$ (see Figure 1b for an illustration). In other words, $\tau^*(\eta)$ is a decreasing function of η so that *the set of transport values for which agglomeration is preferred to dispersion monotonically shrinks as the degree of aversion to inequality rises.*

According to the utilitarian approach ($\eta = 0$), agglomeration is always preferred to dispersion whenever the elasticity of substitution is sufficiently low; otherwise, agglomeration remains socially desirable once transport costs are below to the threshold $\tau^*(0)$. When equity becomes a more important societal concern, that is, when η rises, agglomeration is preferred to dispersion over a narrower range of transport cost values. Finally, as expected, dispersion is desirable for all levels of transport costs and all values of the elasticity of substitution when the SWF is Rawlsian ($\eta \to \infty$).

So far, we have not investigated the connections between our various welfare approaches and the market outcome. We show below how this can be accomplished by focusing on particular values of η.

4.2 Bentham, Kaldor and Rawls

Consider first the SWF with the value $\eta = 2$. Unexpectedly, the function $F(\tau)$ defined in (18) is identical to $\Delta(\tau; 2)$ because $1-\eta = -1$.[13] This means that the threshold obtained by using Kaldor's compensation criterion, τ_K, is identical to $\tau^*(2)$. Put differently, *the solution obtained under Kaldor's criterion is identical to the one with the SWF* (19) *in which the degree of aversion toward inequality is equal to* 2.

Consider next the case where $\eta = 0$, namely the SWF is given by the utilitarian criterion. To start with, note that agglomeration is always desirable from the utilitarian standpoint once $\sigma < \widehat{\sigma}(0)$. In other words, because $\widehat{\sigma}(0) > \overline{\sigma}$, the market outcome is always optimal from the utilitarian standpoint when the black hole condition prevails. Yet, we have seen that $\tau < \tau_K$ must hold for agglomeration to be efficient in Kaldorian terms. This suggests that utilitarianism is biased toward agglomeration. To confirm it, we compare the threshold $\tau_u \equiv \tau^*(0)$ to the critical value τ_K obtained by applying Kaldor's compensation mechanism. Because $\tau^*(2) < \tau^*(0)$, we have $\tau_K < \tau_u$, which means that the range over which the utilitarian criterion supports agglomeration is wider than the range obtained by applying Kaldor's compensation principle.

Given the role played by the utilitarian criterion in public economics, it is interesting to assess the performance of the market outcome against it when the black hole condition does not hold. To this end, we must rank the thresholds τ_u, τ_b and τ_s (where τ_b and τ_s are defined in section 2). For $\sigma = \widehat{\sigma}(0)$, we have $\Delta(\tau; 0) \to 0^-$ when $\tau \to \infty$. In this case, it is readily verified that $\tau_b < \tau_s < \tau_u \to \infty$ for all admissible values of μ. By continuity, the same inequalities remain valid when σ slightly exceeds $\widehat{\sigma}$. We know that both τ_b and τ_s decrease when σ rises (Fujita *et al.*, 1999, p. 75), whereas $d\tau_u/d\sigma < 0$ so that all thresholds moves to the left with an increase of the elasticity of substitution. However, simulations reveal that τ_u moves faster than τ_s once σ takes sufficiently large values. The equations being nonlinear, we find it very hard to provide an analytical derivation of these properties. Instead, we present some numerical illustrations. Figure 2a plots τ_u, τ_b and τ_s against μ for $\sigma = 5$, while Figure 2b plots these values against σ for $\mu = 0.7$. On both

[13] In the case $\eta = 2$, the social welfare function is the harmonic mean of the individuals' welfare.

figures, we see that $\tau_b < \tau_u < \tau_s$. This suggests the following results: *the market outcome is desirable from the utilitarian standpoint as long as* $\tau > \tau_s$ *or* $\tau < \tau_b$. When $\tau_b < \tau < \tau_s$, both agglomeration and dispersion are equilibria, but dispersion (resp., agglomeration) is socially desirable if $\tau_u < \tau < \tau_s$ (resp., $\tau_b < \tau < \tau_u$).

Insert Figure 2 about here

Finally, observe that $\tau' = 1$ if and only if $\eta \to \infty$. Hence, for any finite degree of aversion to inequality, there is always a nonempty range of transport costs values for which agglomeration is socially desirable. *It is only in the extreme case of a Rawlsian SWF that dispersion is always preferred to agglomeration.* This amounts to saying that the market outcome is desirable from the Rawlsian standpoint as long as τ exceeds the sustain point τ_s, but yields the opposite outcome once τ is less than the break point τ_b.

5 Concluding remarks

Using Krugman's core-periphery model, we have investigated whether there are conditions under which agglomeration is socially better than dispersion, and vice versa. Our results show that there is no simple answer to this question. However, two main conclusions emerge from our analysis: (i) there is indetermination about the superiority of one configuration over the other when transport costs are not very low and (ii) the utilitarian criterion provides a very crude approximation of the efficient outcome. Whereas the former may explain *a posteriori* the reticence of some scholars to perform welfare analyses in economic geography models, the latter invites us to be more careful about recommendations based on the maximization of a simple utilitarian welfare function considered as a measure of efficiency. Furthermore, the use of social welfare functions exhibiting different attitudes toward inequality reveals that the evaluation of the market outcome heavily depends on societal values. In particular, our analysis confirms the existence of a trade-off between efficiency and equity. Because we may reasonably expect these results to hold true in more general and realistic models, it will likely be hard to make any strong policy recommendation regarding the spatial distribution of economic activities at the interregional level.

As a last comment, note that, although we have followed a different research strategy, our welfare analysis points to the same direction as Ottaviano and Thisse (2002). This confirms the idea that both settings (Krugman, 1991; Ottaviano *et al.*, 2002) belong to a broader class of economic geography models sharing similar properties. The study of this class of models should be given more attention in future research.

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Appendix

$$F(\tau) \equiv \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{-\mu}{\sigma-1}} - \left(\frac{1+\mu}{2} + \frac{1-\mu}{2}\tau^{\mu}\right)$$

Set

$$\phi \equiv \tau^{1-\sigma}$$

Then, $F(\tau)$ can be rewritten as follows:

$$f(\phi) \equiv \left(\frac{1+\phi}{2}\right)^{\frac{\mu}{1-\sigma}} - \left(\frac{1+\mu}{2} + \frac{1-\mu}{2}\phi^{\frac{\mu}{1-\sigma}}\right)$$

which is defined on $(0,1)$. Note first that $f(1) = 0$ whereas $f(\phi) \to -\infty$ when $\phi \to 0$. It is readily verified that $f(\phi)$ has a single extremum at

$$\phi^* = \frac{1}{2(1-\mu)^{-\frac{\sigma-1}{\sigma-1+\mu}} - 1}$$

which can be shown to belong to $(0,1)$. Because

$$\lim_{\phi\to 1} f'(\phi) = \frac{\mu^2}{2(1-\sigma)} < 0$$

$f(\phi)$ is decreasing in the left neighborhood of $\phi = 1$. This in turn implies that, as ϕ varies from 1 to 0, $f(\phi)$ first increases, reaches its maximum at ϕ^* and, finally, decreases without bound. As a result, $f(\phi) = 1$ for a single value $\phi_K < 1$ such that $f(\phi) > 0$ if and only if $\phi > \phi_K$. Let τ_K the value of τ which corresponds to ϕ_K. Hence, $F(\tau) > 0$ if and only if $\tau < \tau_K$.

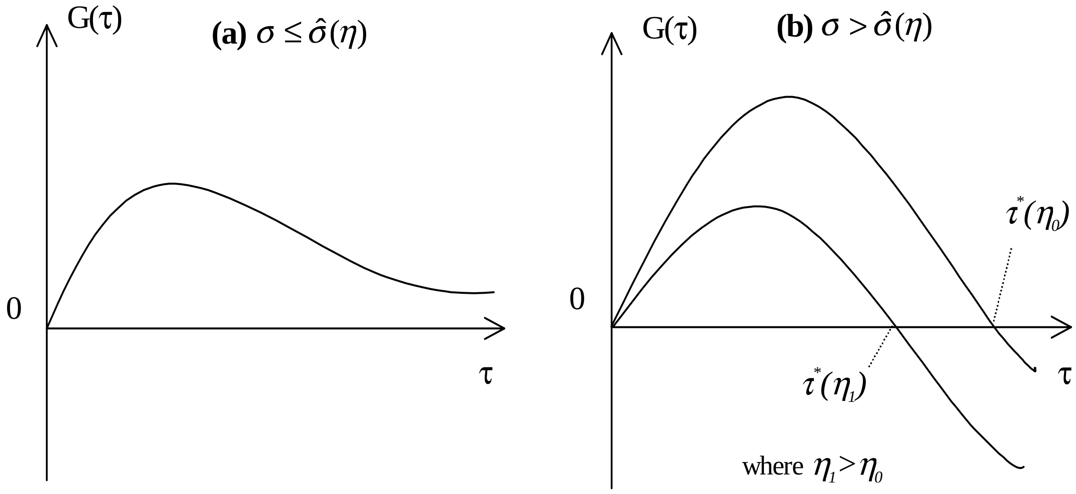

Figure 1.

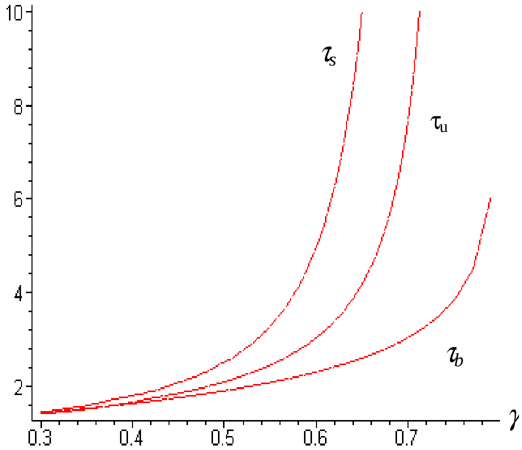

Figure 2a. $(\sigma = 5)$

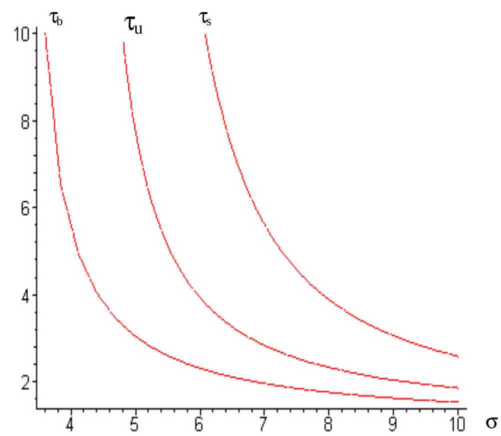

Figure 2b. $(\mu = 0.7)$